Adaptability, productivity, and educational incentives in a matching model*

Olivier CHARLOT†, Bruno DECREUSE‡ and Pierre GRANIER§

Abstract: We study the connections between the labour market and the education sector in a matching framework with ex-post wage bargaining. Workers have multidimensional skills and the search market is segmented by technology. Education is a time-consuming activity and determines jointly the scope – or adaptability – and intensity – or productivity – of individual skills. We establish three main results. First, unemployment provides incentives to schooling by raising the need for adaptability. Second, private returns to productivity are below social returns, but no hold-up phenomenon is involved. Third, due to wage and congestion externalities, private returns to adaptability exceed social returns. As a consequence, both over- and under-education may take place in equilibrium.

Keywords Matching frictions; Returns to education; Human capital

J.E.L. classification: I20, J21, J60

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*This paper has benefited from comments of participants to seminars in EUREQua and GREQAM, and to the EEA and ESEM conferences of August 2001 in Lausanne. We are also indebted to three referees and an editor of this review. The usual disclaimer applies.

†GREQAM – 2 rue de la Charité, 13002 Marseille, France. E-mail: charlot@ehess.cnrs-mrs.fr

‡EUREQua, CNRS and Université Paris 1 Panthéon Sorbonne – Maison des Sciences Economiques, 106-112 boulevard de l'Hôpital, 75647 Paris cedex 13, France. E-mail: decreuse@univ-paris1.fr

§Université de la Méditerranée, GREQAM and IDEP – 2 rue de la Charité, 13002 Marseille, France. E-mail: granier@ehess.cnrs-mrs.fr
1 Introduction

Is the educational level resulting from the market incentives socially efficient? Since Becker, it is well-known that the market outcome is socially optimal provided the labour market is perfectly competitive: workers bear the full cost of their education, and are rewarded their full marginal productivity. However, this cannot be so in the presence of unemployment, which creates important externalities and gives rise to wage bargaining: this is exemplified by the contributions by Laing et al (1995) and Acemoglu (1996) analyzing matching models with *ex post* Nash bargaining and *ex ante* costly human capital investments\(^1\). In this setup, it is established that workers under-invest in education. This is the result of a hold-up phenomenon: as wages are determined by *ex-post* rent sharing between firms and workers, workers carry the full marginal cost of their education, but only get a share of the marginal reward\(^2\).

However, these latter contributions predict that unemployment is detrimental to human capital investment, which fails to square with a major feature of the European labour market where a simultaneous rise in the duration of studies and in unemployment has been observed during the 70’s and the 80’s\(^3\).

The theoretical negative relationship between unemployment and education hinges on two main assumptions: (i) education is costly but not time consuming (e.g. tuition fees); (ii) education improves workers’ productivity but does not alter their chance of finding a job. The first assumption implies that the cost of education does not depend on the state of the labour market (e.g. the level of unemployment). Since the sole aim of educational investments is to raise a worker’s productivity and future earnings, the consequence of the second assumption is that a high unemployment rate is detrimental to human capital investments: when unemployment is high, a worker’s human capital remains idle a larger proportion of the time, which depresses the return to education.

In this paper we alter these two assumptions. We consider a matching model with *ex-post* Nash bargaining, where workers have multidimensional skills and the search market is segmented by technology. Education is a time-consuming activity which involves no other cost than foregone earnings and determines jointly the scope of a worker’s skills – *i.e.* the number of technologies a worker can operate, which we also term adaptability – and the intensity of those skills – that is the worker’s productivity on the various technologies she can operate. Provided ads for jobs convey sufficient information about the type of skills

\(^1\)See also Masters (1998) and Burdett and Smith (2001).

\(^2\)Search frictions play an important role in that result. In a frictionless environment with two-sided investment, Cole et al (2001) show that there can also be over-investment.

\(^3\)In the appendix, we provide some basic figures documenting the rise in school life expectancy, school enrollment rates and standardized unemployment for several European countries. See also Manacorda and Petrongolo (1999) for additional evidence. The simultaneous rise in education and unemployment may suggest that the incentives to schooling may increase with unemployment: Petrongolo and San Segundo (2002) estimate the impact of unemployment on the demand for education on a Spanish dataset, and show evidence of a positive effect of youth unemployment on the demand for education.
needed to perform the job, we can assume that workers direct their search towards the market segments where they have the relevant skills. As a consequence, a higher educated has more chance of leaving unemployment, as she can apply to a larger fraction of jobs in the economy, \textit{i.e.} investing in education yields employment returns as the higher educated benefit from a higher contact rate\textsuperscript{4}. Alternatively, workers may not direct their search towards the jobs they are suited for when the information on the skill requirements is too poor. In this case, all workers have the same contact rate (whatever their education level) but higher educated workers still exit faster from unemployment as they can form matches with more firms. This alternative specification is examined in section 5.

In this setup, our main findings can be depicted as follows: assume first that the sole benefit of education is a higher productivity. Since the only cost involved during the schooling period is an opportunity cost, the shadow price of education is always proportional to the reward. This implies that the amount invested in education is independent of both the state of the labour market (unemployment) and wage setting parameters (workers’ bargaining power). As in Acemoglu (1996), workers under-invest in education but this does not rest on a hold-up argument. This is due to a positive externality originating from the effect of education on productivity. A more productive workforce raises firms’ incentives to entry, which in turn raises employment opportunities for all workers.

Assume now that education only improves the scope of an individual’s skills. The incentives to education increase with job scarcity and decrease with average employment duration, the two main determinants of unemployment. This can be so because low job creation and/or high job destruction reduce the opportunity cost of an additional schooling period, while it raises the benefits. In addition, private returns to schooling exceed social returns and workers over-educate. This is due to the combination of two effects. First, education not only improves the exit rate from unemployment, but can also be used to raise one’s outside opportunities during the wage bargain\textsuperscript{5}. The former effect is met by a social gain, as education reduces aggregate unemployment, but the latter is not. Second, education raises congestion effects between job-seekers, which reduces the social benefit from adaptability. Taken together, these two effects dominate the positive externality mentioned above and over-education takes place\textsuperscript{6}.

\textsuperscript{4}Note that workers have the same search intensity on all the markets where they participate, and that this search intensity does not decline with the range of prospected markets. This could also be so if search intensity were a choice variable, under the assumptions of decreasing returns to search in each market, and of a linear search cost. Decreasing marginal returns to search imply that workers set the same search intensity on each market, while a linear search cost guarantees the average search intensity does not decline as the number of prospected markets increases.

\textsuperscript{5}In a nutshell, consider there are only two types of jobs in the economy, \textit{e.g.} jobs where either German or Spanish is needed as a foreign language. Those who speak two languages (\textit{e.g.} College graduates) have twice as many job opportunities as those who speak only one language (High school graduates), and earn higher wages as they have better outside opportunities.

\textsuperscript{6}The alternative version of our model studied in section 5 helps to make it clear that the over-education result is not due to the existence of employment returns to schooling, but to the combination of wage and congestion externalities. When the search market is unsegmented, workers compete for all jobs regardless
In the general case where education affects both adaptability and productivity, private returns to schooling may either be larger or lower than social returns. This generally depends on the technologies relating education to adaptability and productivity, the rate of entry in the schooling system, the matching technology, and the workers’ bargaining power. The efficient allocation can be decentralized by means of a combination of labour market and education policies.

The two dimensions of education highlighted in this paper can be thought of as two polar cases, e.g. general vs vocational education. A “purely” vocational or professional education system is meant to improve workers’ efficiency at the workplace, but does not necessarily prepare them to switch from one activity to another over their working life. In a “purely” general education system, students learn how to apply theoretical concepts to a number of issues. This does not necessarily increase their future productivity on a given occupation, but improves their ability to adapt in various types of occupations requiring different knowledge. We argue that both dimensions generally coexist: even in a vocationally or professionally oriented education, the syllabus is comprised of a number of academic courses. In the same way, even a more academic type of education contains vocationally oriented courses (think of compulsory training).

How does education determine the scope of a worker’s skills? The underlying idea in our model is that students acquire a number of abilities in various fields during their schooling, the number of which increases with the schooling duration. Any production process or technology is defined as a given combination of abilities; a worker can operate a technology provided she embodies the required combination. As the number of abilities increases with the duration of schooling, the higher educated can combine these abilities in a larger number of ways, and can therefore perform a larger number of jobs.

This view of adaptability comes close to Nelson and Phelps’ (1966), who argue that one of the major benefits of education is that the workforce can adapt more rapidly to technological change. We provide microfoundations for this type of benefit from education, and investigate its macroeconomic implications.

The notions of adaptability and productivity studied in this paper can be paralleled with the notions of marketability and specialization highlighted in the literature on money and search (see e.g. Kiyotaki and Wright, 1993, and Shi, 1997). The main idea in these papers is that each producer faces a trade-off between specialization and adaptability. Specializing in the production of a given commodity allows better productivity (or, equivalently, saves on production costs), but at the expense of a lower proportion of consumers interested in purchasing the good, i.e. less marketability that is the size of their potential of the technologies they embody. Congestion effects cannot increase with education, as they are already maximized. In such a case, private returns to schooling may be lower than social returns, even if education does not affect productivity.

We leave for further research the issue of how workers would respond to technological change in our setting.
market. Typically, money plays a crucial role in this approach as it enlarges the size of the market and therefore allows producers to specialize. In our paper, there is no explicit trade-off between adaptability and productivity: by educating longer, individuals improve both adaptability and productivity.

The closest paper to ours is that by Moen (1999). Though the two papers share some common theoretical predictions, two main differences are worth emphasizing, regarding the modelling of the matching process and the cost of education. Moen considers an urn-ball matching process, where firms may receive multiple simultaneous applications for a vacant position. The applicants are ranked according to their education level, and the most productive worker is hired. In such a setting, workers invest in education not only to get higher earnings, but also to improve their ranking in the job queue, i.e. there are private employment returns to schooling. Importantly, the number of matches at the aggregate level is independent of the education level of the workforce, so that there are no social employment returns to schooling. Moen shows this may dominate the hold-up phenomenon and workers may over-educate. By contrast, no hold-up is involved in our paper, as students do not bear any other cost thanforegone earnings during their studies. However, workers may under-invest in education due to the positive externality on firms’ incentives to enter the search market. Additionally, there are social employment returns to schooling as education makes the search process more efficient, but over-education may still occur, resulting from wage and congestion externalities.

Signalling models (Arrow, 1973, Spence, 1973) offer a competing explanation for over-investment in education. Our paper shares a theoretical prediction with these models: a worker’s wage can be positively related to her education level, even if education does not alter one’s productivity. However, important differences can also be noticed. In signalling models, social returns to schooling differ from private returns since one of the function of education is to screen individuals. If there were no social benefits from screening, over-education could take place. However, signalling may feature social returns if, for example, workers and tasks were better matched as a result of screening (see e.g. Stiglitz, 1975). As screening eliminates the wage subsidy between individuals which is relevant at the private but not at the social level, there can be too little or too much screening. In our

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8 This is so in Shi’s paper. In Kiyotaki and Wright (ibid), the trade-off is actually different as “specialization” offers a higher rate of contact of consumers, but reduces the probability a given consumer is willing to purchase the good, that is “marketability”.

9 This mechanism refers to Thurow’s (1975), who argues workers are ranked in a job queue according to their training cost. Higher educated have lower training cost, and are thus ranked higher in the job queue. See also Brunello and Medio (2001).

10 This may also be the case in Saint-Paul (1996). In a dual matching model of the labour market where firms allocate a fixed number of job slots between two sectors, education features private employment returns, as unemployment is lower in the high-skill sector. However, there may be some excess supply of skills, as a rise in the number of educated workers causes a rise in unemployment both for educated and uneducated workers. Obviously, this negative externality collapses when the number of vacancies is determined by free entry.
paper, over-education does not rely on unobserved heterogeneity and on the capture of an ability rent: workers acquire some excess education to improve their opportunities during the wage bargain, while those skills are genuinely aimed at raising their chance of leaving unemployment.

In a different setup, Cole et al (2001) have already highlighted inefficient investments: the authors consider a frictionless framework with two-sided heterogeneity, ex ante investments and ex post bargaining. As in our paper, private investments alter both the size of the surplus to be shared and each party’s outside options during the bargain, but this stems from two-sided heterogeneity on the market rather than from a higher contact rate.

We are aware of three other theoretical papers dealing with multidimensional skills in a matching framework. Moscarini (2001) introduces search frictions in a Roy (1951) type of model of self-selection between sectors on the basis of comparative advantage. Marimon and Zilibotti (1999), and Barlevy (2002) assume workers and firms are exogenously located on a circle representing the technological space. The longer the distance between a worker and a firm, the lower the productivity of the pair. All these papers consider skills as given. Therefore, we make a contribution to this literature by analyzing education investments.

The paper is organized as follows: section 2 describes the basic environment and the partial equilibrium of the labour market. Section 3 is devoted to the analysis of the general equilibrium. Section 4 deals with social efficiency. In section 5, we consider an alternative specification of the search process where the matching market is unsegmented. Section 6 concludes.

2 The model

Time is continuous. There is a large number of small firms, the number of which is endogenous in equilibrium. Each firm is endowed with a single vacancy which can be vacant or filled. There exists a continuum of different but equally productive technologies producing the same good. Each firm embodies a technology which is randomly drawn from a uniform distribution on the interval [0, 1].

At each instant, $\delta > 0$ agents are born students. Each individual is facing a constant risk of dying $\delta$, so that the population is constant and normalized to unity. Agents are risk-neutral and $\rho$ is the pure rate of time preference, as well as the interest rate of the economy; $r \equiv \rho + \delta$ is thus the global discount rate. The economy is comprised of two different theatres of activity: an education sector and the labour market.

*Education sector.* The agents determine the length of their studies in response to the

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11 Leaving aside the productivity component of schooling, our technology can be seen as a particular case of Marimon and Zilibotti’s: below a critical distance the pair is fully productive; above, the match is totally inefficient. The key point here is that education lengthens the critical distance.
incentives provided by the market. The aim of education is twofold: it increases the scope of workers’ skills, and their productivity. The individual duration of studies \( T_i \) determines the number of technologies \( F(T_i) \) a worker can operate\(^{12}\). This is the **extensive effect of education**, which raises the workers’ job-finding rate via a better mobility on the matching market.

In addition, education increases workers’ productivity, as usual in the human capital theory (Becker, 1964, Laing, Palivos and Wang, 1995): this is the **intensive effect of education**, which raises workers’ productivity on a given occupation.

**Assumption 1 Education, adaptability and productivity**

A1. The function \( F: [0, +\infty) \rightarrow [0, 1] \), (i) is strictly increasing, strictly concave and twice continuously differentiable, (ii) satisfies the boundary conditions: \( F(0) = 0 \) and \( \lim_{x \to \infty} F(x) = 1 \).

A2. The function \( y: [0, +\infty) \rightarrow [0, +\infty] \), (i) is strictly increasing, strictly concave and twice continuously differentiable, (ii) satisfies the Inada conditions and the boundary condition: \( y(0) = 0 \).

For analytical convenience, let us define the remainder \( \lambda_y(T) \equiv y'(T)/y(T) \) and \( \lambda_F(T) \equiv F'(T)/F(T) \). The variable \( \lambda_y \) is the part of the marginal return to education associated with an enhanced productivity, while \( \lambda_F \) is the part of the marginal return associated with a higher matching rate.

**Matching sector:** After their schooling period, the agents enter the labour market as unemployed, and try to locate a vacancy. The search market is segmented by technology: there is a separate search sub-market for each technology. Until section 5, we assume job advertisements convey full information on the skill requirement and workers only apply on the sub-markets where they have the required skills\(^{13}\). In section 5, we suppose the information on the skill requirement is too poor, and workers apply on all sub-markets, regardless of the embodied technology\(^{14}\). On each sub-market \( j \), the flow of matches \( M_j \) is given by the matching technology:

\[
M_j \equiv M(n_j, v_j)
\]

where \( n_j \) and \( v_j \) are (resp.) the number of job-seekers and vacancies on market \( j \). The function \( M \) has the following properties.

\(^{12}\)More formally, let \( F_{T_i} \subset [0,1] \) be the subset of technologies the worker can operate. The function \( F(T_i) \) is the Lebesgue measure of \( F_{T_i} \).

\(^{13}\)It is thus implicitly assumed that participating to a given search sub-market involves an epsilon-cost, which prevents workers from sending applications to positions they are unable to occupy. Alternatively, firms may in a first step sort workers’ applications towards positions they are suited for.

\(^{14}\)In the former case, raising one’s schooling duration creates congestion effects for the other workers, but there is no mismatch in the sense all meetings lead to matches. In the latter case, congestion effects are always maximised, and raising education reduces the mismatch between firms and workers.
**Assumption 2 The matching function**

The function $M : [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ (i) is strictly increasing in each of its arguments, strictly concave and admits constant returns to scale, (ii) satisfies the following boundary conditions: $M(z,0) = M(0,z) = 0$, for $z \geq 0$, and the Inada conditions.

 Matches are equiprobably distributed among job-seekers, as well as among vacancies. Denoting by $\theta_j \equiv v_j/n_j$ the market-specific tightness, the flow probability for a worker to locate a vacancy on market $j$ is $m(\theta_j) \equiv M(1, \theta_j)$. Symmetrically, the flow probability for a firm to meet an applicant is $\eta_j = m(\theta_j)/\theta_j$. The individual job-finding rate is then

$$\mu_i = \int_{j \in F(T_i)} m(\theta_j) \, dj$$  \hspace{1cm} (2)

The law of large numbers implies $v_j = v$ and $n_j = U F$, where $U$ is the mass-number of unemployed and $F \equiv E \{F(T_i)\}$ is the number of technologies which can be operated on average by a “typical” worker. Consequently, market-specific tightness is $\theta_j = \theta$, and contact rates are $\eta_j = \eta(\theta)$ and

$$\mu_i = F(T_i) m(\theta)$$  \hspace{1cm} (3)

The individual job-finding rate is increasing in individual schooling duration. Adaptability returns to schooling will therefore result from a higher contact rate. At the aggregate level, it is worth noting that the efficiency of the matching process is increasing in workers’ education. This can also be seen by considering the aggregate matching function. The aggregate number of matches is $\int_0^1 M_j dj = M(\overline{F}U, v)$. At given number of unemployed, an increase in $\overline{F}$ leads to a rise in the number of matches.

We now turn to agents’ gains.

**Asset values.** Let $q$ be the job separation rate and $\gamma$ be the flow cost of posting a vacancy. Let us also denote $w(T_i)$, $\mu_i$, the wage and job-finding rate for a worker with a schooling duration $T_i$. The values of being employed and unemployed respectively are $W^e(T_i)$ and $W^u(T_i)$; the values of a vacant job and of a filled job are $V^v$ and $V^e(T_i)$. Those values satisfy the standard arbitrage equations:

$$r W^u(T_i) = \mu_i [W^e(T_i) - W^u(T_i)]$$  \hspace{1cm} (4)

$$(r + q) W^e(T_i) = w(T_i) + q W^u(T_i)$$  \hspace{1cm} (5)

$$\rho V^v = -\gamma + \eta \{E[T_i V^e(T_i)] - V^v\}$$  \hspace{1cm} (6)

$$(r + q) V^e(T_i) = y(T_i) - w(T_i) + (\delta + q) V^v$$  \hspace{1cm} (7)

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15 In equilibrium, all individuals choose the same schooling duration.

16 At given number of vacancies, the individual contact rate is decreasing with the average schooling duration. This may be interpreted as a job competition phenomenon.
Equation (6) deserves some particular comments: since the education attainment of the incoming worker is *a priori* unknown to the firm advertising a vacancy, the value of a filled job has to be taken with an expectation operator $\mathbb{E}$. Wages are determined by Nash bargaining:

$$\beta [V^e(T_i) - V^v] = (1 - \beta) [W^e(T_i) - W^u(T_i)]$$ (8)

where $\beta \in (0, 1)$ is the worker’s exogenous bargaining power. We assume free entry determines the number of vacancies advertised on the search market. This implies the exhaustion of all rents and drives the value of a vacancy down to zero. This yields

$$\mathbb{E}_T V^e(T_i) = \gamma / \eta(\theta)$$ (9)

Free entry implies the expected return $\mathbb{E}_T V^e(T_i)$ to a job is equal to the expected search cost $\gamma / \eta$.

### 3 Education and the labour market performance

In this section we focus on the properties of the equilibrium. We first describe the partial equilibrium, *i.e.* education behaviours for a given labour market tightness and the labour demand for a given schooling duration. We then compute the general equilibrium and highlight its main characteristics.

**Individual schooling duration.** Wages can be derived from equation (4) to (8):

$$w(T_i) = \frac{\beta (r + q + F(T_i) m(\theta))}{r + q + \beta F(T_i) m(\theta)} y(T_i)$$ (10)

The value of search is

$$W^u(T_i) = \frac{F(T_i) m(\theta)}{r + q + F(T_i) m(\theta)} \frac{w(T_i)}{r}$$

It consists in the discounted value of earnings $w(T_i)/r$ as in a frictionless environment, times a factor $\mu_i/(r + q + \mu_i) < 1$, which takes into account the existence of unemployment and individual time discount. This factor is increasing in the individual job-finding rate $\mu_i$ and decreasing in job destruction rate $q$. It is also decreasing in $r$, because each individual discounts time and starts his/her working life as unemployed.

Optimal schooling maximizes the present-discounted value of search

$$\hat{T} \in \arg \max_{T \geq 0} e^{-rT} W^u(T)$$ (11)

The first order condition writes down $MC^p \equiv r W^u(\hat{T}) = dW^u(\hat{T})/dT \equiv MR^p$. The left-hand side is the private marginal cost of education, which consists in the value of
foregone earnings during an additional year of schooling. The right-hand side stands for
the private marginal return. It can be decomposed as follows:

\[
MR^p = W^u \left\{ \frac{r + q}{r + q + \mu} \lambda_F + \lambda_y + (1 - \beta) \frac{r + q}{r + q + \beta \mu} \frac{\mu}{r + q + \mu} \lambda_F \right\} 
\]  

(12)

There are three types of returns to schooling: an employment return, as education raises
one’s probability of finding a job; an intensive wage return as education raises productivity;
an extensive wage return, as education improves outside opportunities during the wage
bargain. The existence of this latter type of return implies workers invest in skills genuinely
designed to exit more rapidly from unemployment, but with the aim of bargaining a higher
wage once employed.

Proposition 1 A characterization of the optimal schooling duration

(i) The optimal schooling duration \( \hat{T} \) solves

\[
\lambda_y \left( \frac{\hat{T}}{r} \right) + \lambda_F \left( \frac{\hat{T}}{r + q + \beta \mu} \right) m(\theta) = r 
\]  

(13)

(ii) \( \hat{T} \) is increasing in \( q \), and decreasing in \( \theta \) and \( \beta \).

At given tightness, all individuals choose the same schooling duration\(^{17}\). It is increasing
in the job destruction rate \( q \), decreasing in tightness \( \theta \) and workers’ bargaining power \( \beta \).
These properties directly stem from the assumption that education takes time, and not
only affects productivity, but also the scope of an individual’s skills. We now develop
these points in detail.

Assume first that \( \lambda_F = 0 \). Then one gets a standard model of human capital investment
where workers invest in education to raise their productivity. An important feature of this
model is that education takes time. Therefore the cost of education is endogenous and
-corresponds to the value of foregone earnings during the schooling period. This implies
that the shadow price of education is always proportional to its reward and education does
not depend on variables characterizing the working of the labour market; e.g. neither on
workers’ bargaining power, nor on labour market tightness. This property differs from
that obtained when schooling involves some direct costs, such as tuition fees: in this case
workers are held up, as they bear alone the full marginal cost of their education, but wage
bargaining implies they do not get the full marginal reward of it (see e.g. Acemoglu,
1996). This feature has important implications in terms of welfare (see section 4): if
workers under-invest in education here, this has nothing to do with a hold-up problem.

\(^{17}\) Equation (13) is a generalization of human capital earnings functions such as those surveyed by Willis
(1986).
Note that we assume the unemployed do not receive any income during search. One may wonder if the latter result would still hold when such an income were introduced. If that income were proportional to productivity (because, say, unemployment benefits are proportional to past earnings, or because the value of leisure increases with education), then our conclusion would remain: the return to search would be proportional to productivity, and so would the shadow cost of education. Conversely, if the income during search were not proportional to productivity, then the shadow cost of education would not be proportional to its reward. It follows optimal schooling would decrease with search income, and increase with worker’s bargaining power and job-finding rate – the usual properties when education involves monetary costs\(^{18}\).

Conversely, suppose \(\lambda_y = 0\). Workers invest in education to improve their job-finding rate while unemployed, and their outside opportunities while employed. As the share of the surplus obtained by a worker is increasing in her bargaining power, there are less incentives to invest in education to raise one’s outside opportunities when the bargaining power is already large. An increase in the job destruction rate raises the return to education, as it becomes relatively more important to improve one’s chance of finding a job when the frequency of unemployment spells increases. For similar reasons, a drop in labour market tightness increases the return to education, since it is relatively more important to raise one’s job-finding rate when the market is loose.

We now characterize the labour market tightness \(\theta\) for a given education attainment \(T\).

*Labour demand and education.* Free entry and equations (6)-(7) imply

\[
\frac{\gamma}{\eta(\theta)} = \frac{(1 - \beta)}{r + q + \beta F(T)m(\theta)} y(T) \tag{14}
\]

**Proposition 2** A characterization of labour market Tightness

(i) There exists a unique labour market tightness \(\hat{\theta} > 0\) solving (14)

(ii) \(\hat{\theta}\) is decreasing in \(q\) and in \(\beta\)

(iii) The effect of \(T\) on \(\hat{\theta}\) has the sign of \(\lambda_y - \frac{\beta \mu}{r + q + \beta \mu} \lambda_F\); i.e. \(\text{sign}\left\{\frac{d\hat{\theta}}{dT}\right\} = \text{sign}\left\{\lambda_y - \frac{\beta \mu}{r + q + \beta \mu} \lambda_F\right\}\)

The effects of workers’ bargaining power, discount rate \(r\), job destruction rate \(q\) and advertising cost \(\gamma\) are usual: they all reduce the profitability of holding a vacancy, and cause a fall in tightness.

The effects of a rise in schooling duration are twofold. On the one hand, there is a positive productivity effect, which raises the size of the surplus to be shared between firm and worker. This effect increases the profitability of a vacancy and thus tends to

\(^{18}\)Let \(b\) denote the (productivity-independent) unemployment income. In an interior solution, optimal schooling is given by: \(\lambda(T) = r + rb/\left(\beta \mu y(T)/r\right)\).
raise tightness. On the other hand, there is a negative wage effect, which reduces the profitability of a vacancy. A longer schooling duration improves a worker’s chance of finding a job, and thus improves her outside options during the wage bargain. As the search market is segmented by technology, this effect is not matched by a corresponding increase in the contact rate for vacancies. The wage effect tends to lower tightness. The global impact of education on tightness is ambiguous; it has the sign of \( \lambda_y - \frac{\beta \mu}{r+q+\beta \mu} \lambda_F \).

Population. Once tightness and schooling duration are known, we can compute the stationary populations in each state (education, unemployment, employment). The mass number of workers enrolled in the education system is then \( S = \int_0^T \delta e^{-\delta z} dz = 1 - e^{-\delta T} \). Therefore, the measure of the working population is equal to the participation rate to the labour market \( e^{-\delta T} \). From the flow equilibrium, we deduce the unemployment rate \( u = \frac{\delta + q}{r+q+\mu} \). The mass-number of unemployed is \( U = ue^{-\delta T} \), while employment is \( L = (1-u)e^{-\delta T} \).

Equilibrium analysis. An equilibrium is a pair \((T^*, \theta^*)\) such that

\[
\lambda_y(T^*) + \lambda_F(T^*) \frac{r+q}{r+q+\beta F(T^*) m(\theta^*)} = r \quad \text{(OS)}
\]

\[
\frac{\gamma}{\eta(\theta^*)} = \frac{(1-\beta)y(T^*)}{r+q+\beta F(T^*) m(\theta^*)} \quad \text{(FE)}
\]

**Proposition 3 Existence and characterization of equilibrium**

(i) There exists a unique equilibrium.

(ii) The tightness is decreasing in \( \beta \). The effect of \( \beta \) on the schooling duration has the sign of \( \beta + \alpha^* - 1 \), i.e. \( \text{sign} \{dT^*/d\beta \} = \text{sign} \{\beta + \alpha^* - 1\} \), where \( \alpha^* \equiv \theta^* m'(\theta^*)/m(\theta^*) \)

(iii) The schooling duration is increasing in \( q \). The effect of \( q \) on tightness is negative whenever \( \lambda_F^* \geq \lambda_y^* \); otherwise it is ambiguous.

A standard property of matching models with human capital investment is that there are aggregate increasing returns to education: a more productive workforce raises the labour demand. Conversely a tighter labour market increases the incentives to schooling. This mechanism can give rise to multiple equilibria (see e.g. Laing et al., 1995, and Burdett and Smith, 2001). In our framework, the equilibrium is unique. Uniqueness can easily be understood when the productivity effect of schooling dominates the wage effect (point (iii) of proposition 2) – this is for instance the case whenever \( \lambda_y(T) > \lambda_F(T) \) for all \( T > 0 \). Then, the \textit{optimal schooling} (OS) locus defines education as a decreasing function of tightness, while the \textit{free entry} (FE) locus defines tightness as an increasing function of education. However, uniqueness may seem less obvious when the productivity effect of schooling is dominated by the wage effect. Then both the (OS) and (FE) loci define decreasing relationships between education and tightness. The reason for uniqueness is that workers choose their schooling duration on the basis of the labour demand they face,
while tightness on a particular matching market is only a part of the aggregate labour demand. In this model, the proper measure of the labour demand is the ratio of aggregate number of vacancies per unemployed \( x = \theta F(T) \). But if (sub-)market tightness may decrease with schooling duration, the aggregate number of vacancies \( x \) actually increases with it. Formally, \( dx/dT \) has the sign of \( \lambda F + d\theta/dT > 0 \). At the aggregate level, an increase in schooling duration exerts a positive allocative effect on the probability of matching a given vacancy; this effect dominates the negative wage effect.

The effects of workers’ bargaining power on schooling duration is ambiguous: in partial equilibrium, the bargaining power reduces the incentives to schooling. However, in general equilibrium, it also lowers tightness, which raises incentives to schooling. The former (latter) effect dominates when the worker’s bargaining power is lower (higher) than the elasticity of firms’ recruitment rate with respect to tightness, \( \beta < 1 - \alpha \) (\( \beta > 1 - \alpha \)). For \( \beta = 1 - \alpha \), the worker’s wage is maximized, or equivalently the incentives to education are minimized. Consequently, the equilibrium relationship between education and bargaining power is U-shaped.

An increase in the job destruction rate unambiguously causes a rise in schooling duration, both because of the direct effect of the job destruction rate on the incentive to invest in education (see proposition 1), but also because of its indirect effect on the labour demand, which tends to reduce tightness (see proposition 2). On the contrary, the impact of job destruction on equilibrium labour market tightness is a priori ambiguous: it directly tends to shorten job tenures and depresses the labour demand, but for similar and opposite reasons, it also raises the schooling duration and the productivity of the workforce.

4 Over versus under-education

In this section, we compare the market outcome to the social commands of a benevolent central planner. Private and social schooling costs are very similar. However, due to wage and congestion externalities, the private and social returns to schooling generally differ. Both over- and under-education are plausible outcomes as a result. We also show the social optimum can be decentralized by mean of an adequate policy.

Social commands. In order to avoid some tedious considerations, we consider the case\(^{19}\) where \( \rho = 0 \). The social planner aims to maximize stationary consumption \( C \), i.e. total output net of search costs (see e.g. Pissarides, 1990, and Hosios, 1990 or also Acemoglu and Shimer, 1999):

\[
C = Y - \gamma v
\]

(15)

where \( Y = e^{-\delta T} (1 - u) y \) is gross output, that is participation times one minus unemployment rate, times flow output. The planner controls tightness \( \theta \), and the school-

\(^{19}\)Our perpetual youth assumption is here crucial: if workers were infinitely lived, the maximization problem would not make sense.
Proposition 4 The social optimum

The planner’s commands \((T^s, \theta^s)\) are characterized by

\[
\lambda_y^s + \frac{(1 - \alpha^s)(\delta + q)}{\delta + q + (1 - \alpha^s) F(T^s)m(\theta^s)} \lambda_F^s = \delta \left(1 - \frac{\alpha^s(\delta + q)}{\delta + q + (1 - \alpha^s) F(T^s)m(\theta^s)}\right)
\]

\[
\gamma/\eta(\theta^s) = \frac{\alpha^s y(T^s)}{\delta + q + (1 - \alpha^s) F(T^s)m(\theta^s)}
\]

where \(\lambda_y^s \equiv \lambda_y(T^s), \lambda_F^s \equiv \lambda_F(T^s), \text{ and } \alpha^s \equiv \theta^s m'(\theta^s)/m(\theta^s)\).

While choosing tightness and schooling duration, the planner takes into account changes in gross output and search spendings. The first-order condition with respect to \(\theta\) is

\[
-\frac{du/d\theta}{1-u} Y - \gamma v \left[\frac{1}{\theta^s} + \frac{du/d\theta}{u}\right] = 0
\]

The gain is a lower unemployment rate, which raises gross output. The cost is a rise in search spendings, due to the direct increase in tightness minus the induced reduction in unemployment.

The first-order condition with respect to \(T\) is

\[
Y \left[-\delta - \frac{du/dT}{1-u} + \lambda_y\right] - \gamma v \left[-\delta + \lambda_F + \frac{du/dT}{u}\right] = 0
\]

A rise in the schooling duration \(T\) affects gross output in the following way: it reduces the participation to the labour market at rate \(\delta\), the rate of entry into the labour force. It also lowers the unemployment rate by improving the efficiency of the search process. Finally, it raises productivity. The rise in schooling duration has also an impact on the amount of search spendings. The schooling duration alters the congestion effects between job-seekers in different and opposite ways. First, the fall in participation reduces congestion externalities for a given number of vacancies. As a consequence, search costs go down at rate \(\delta\). Second, the rise in the scope of the workers’ skills increases congestion effects, and search costs go up at rate \(\lambda_F\). Importantly, these two effects evolve in opposite directions and exactly offset each other whenever \(\lambda_F = \delta\). Third, the fall in unemployment reduces congestion, which contributes to lower search spendings.

Private vs social returns to schooling. Equations (OS) and (FE) are now compared to equations (16) and (17).

Proposition 5 Efficiency properties of search equilibrium

Let \(\lambda_F^s \equiv \lambda_F(T^s)\).
(i) if \( \lambda_F^s > \delta \), there is over-education

(ii) if \( \lambda_F^s = \delta \), there is over-education whenever \( \beta \neq 1 - \alpha^s \)

(iii) if \( \lambda_F^s < \delta \), there exist \( \beta_- \) and \( \beta_+ \), \( \beta_- < 1 - \alpha^s < \beta_+ \), such that there is under-education whenever \( \beta \in (\beta_-, \beta_+) \)

Private and social returns to schooling may differ because of matching-specific and education-specific distortions. Considering equations (FE) and (17), it turns out that the working of the labour market is inefficient at given schooling duration, unless the so-called Hosios condition is satisfied, that is unless \( \beta = 1 - \alpha^s \). However, the Hosios condition does not guarantee private and social returns to schooling coincide. From equation (OS) and (16), social and private schooling durations coincide at given tightness and under the Hosios condition only if \( \lambda_F^s = \delta \), that is the social rate of return to workers’ adaptability is equal to the rate of entry into the labour force. Consequently, the decentralized outcome may be socially efficient, but this is rather an unlikely situation. Indeed, it requires both \( \beta = 1 - \alpha^s \) and \( \lambda_F^s = \delta \).

Therefore, the decentralized outcome is most of the time socially inefficient. More importantly, both over- and under-education may arise. To see this, let us compare the public and private determinations of education:

\[
\delta = u^s \lambda_F^s + \lambda_y^s + \alpha^s \frac{\delta + q}{\delta + q + (1 - \alpha^s) \mu^s} \left[ (\delta - \lambda_F^s) + (1 - u^s) \lambda_F^s \right] \tag{20}
\]

\[
\delta = u^s \lambda_F^s + \lambda_y^s + (1 - \beta) \frac{\delta + q}{\delta + q + \beta \mu^s} (1 - u^s) \lambda_F^s \tag{21}
\]

where \( \mu^i \equiv F(T^i) m(\theta^i) \), and \( u^i \equiv (\delta + q) / (\delta + q + \mu^i) \), \( i = s, \ast \). Private and social costs of education are similar. For the social planner, the cost of education is a fall in output, while the private cost consists in foregone earnings during the studies. The private employment and intensive wage returns are matched by equivalent social gains, however private and social benefits differ since there is no extensive wage return at the social level, while workers do not take into account the effect of their education on search costs.

The two first congestion externalities exactly offset each other when \( \lambda_F^s = \delta \). In this case, private and social benefits coincide when the extensive wage return to schooling balances the third congestion externality, that is the reduction in search costs following the decrease in unemployment rate. This can be so if and only if the Hosios condition holds, which corresponds to the case where the private incentives to schooling are minimized. Otherwise, the extensive wage return to schooling is too high and workers overeducate – this is claim (i) of proposition 5. This remark has two consequences. First, if \( \lambda_F^s > \delta \), overeducation always takes place whatever the size of extensive wage returns to schooling – claim (ii) of proposition 5. Second, if \( \lambda_F^s < \delta \), workers undereducate whenever \( \beta \) does

\[\text{The discount rate equals the rate of entry in the workforce.}\]
not differ too much from $1 - \alpha^*$ – claim (iii) of proposition 5. However, overeducation may still result when the bargaining power is either very low, or very large.

Figures 1a and 1b illustrate the various cases.

Let us focus on two polar cases, corresponding to the two dimensions of education highlighted in this paper.

Suppose first education does not affect the scope of a worker’s skills, that is $\lambda_F = 0$. Therefore we get

$$\lambda_y^* = \delta \left(1 - \alpha \frac{q + \delta}{(1 - \alpha) \mu^y + q + \delta}\right) < \delta = \lambda_y^*$$

This implies that $T^* < T^*$. As in standard matching models with human capital investment, the existence of labour market imperfections translates into under-investment in education. But here, this property does not hinge on a hold-up argument since no direct schooling cost is involved. As stated above by proposition 1, the decentralized duration of studies is not affected by tightness or by bargaining parameters when $\lambda_F = 0$, since the shadow cost of education is proportional to its benefit. The under-investment in education is due to a participation externality. A longer duration of studies reduces participation and thus congestion effects on the search market.

Conversely, suppose education does not affect workers’ productivity, that is $\lambda_y (T) = 0$. Then,

$$\lambda_F^* = \delta + \frac{\delta}{q + \delta} F (T^*) m (\theta^*) > \delta$$

To be compared to

$$\lambda_F^* = \delta + \beta \frac{\delta}{q + \delta} F (T^*) m (\theta^*)$$

This implies that $T^* > T^*$. Hence, private benefits of schooling are higher than social benefits. This over-education result is due to the combination of wage and congestion externalities. The mechanism is the following: workers educate partly to raise their outside opportunities while bargaining. This private return is not met by a social gain. It is responsible for excess job competition between the unemployed and makes the participation rate of the economy too low. Saint-Paul (1996) and Moen (1999) have already elaborated on some undesirable consequences of education in a matching model. In each of these contributions, a worker’s decision to invest in education can be an important source of inefficiency since one’s decision to invest in human capital reduces the others’ chance of getting a job. Put otherwise, schooling features private employment returns, but there is no such social return. In our paper, there are social employment returns to schooling. However, we shed light on another source of inefficiency due to the extensive effect of education on wages.
Another way of highlighting the distortions induced by private schooling decisions is the following. Aggregate stationary consumption can be written as

\[ C = e^{-\delta T} (1 - u) (y - (\delta + q) \gamma / \eta) \] (25)

The amount of resources left for consumption is aggregate employment \( e^{-\delta T} (1 - u) \) (participation times one minus the unemployment rate), times output flow \( y \) minus search spendings \((\delta + q) \gamma / \eta\) per employee. The latter term defines an implicit wage. It is then clear that the planner’s objective coincides with a newborn individual’s welfare. When the Hosios condition holds, social commands can therefore be deduced from\(^{21}\)

\[ \max_T e^{-\delta T} W^u (T, \theta) \] (26)

subject to the decentralized determination of tightness (15) when \( \beta = 1 - \alpha^s \). The first-order condition can be written as follows:

\[ MC^s \equiv \delta W^u (T^*) = MR^p + \frac{\partial W^u}{\partial \theta} \frac{d \theta}{dT} \equiv MR^s \] (27)

Private and social marginal schooling costs coincide. The social marginal return is equal to the private marginal benefit, plus a term accounting for the effect of education on tightness. As \( W^u \) is increasing in tightness, private and social marginal returns to schooling only differ in \( d \theta /dT \). Below proposition 2, we discuss the impact of education on tightness. A positive externality originates from the effect of education on productivity: a highly productive workforce raises the incentives to firms’ entry on the matching market, which in turn raises employment opportunities for all workers. Conversely, the effect of education on adaptability gives rise to a negative wage externality: by increasing the scope of their skills, workers improve their outside opportunities during the wage bargain. This is detrimental to tightness as a result. Point (ii) of proposition 2 shows those externalities exactly offset each other whenever \( \lambda_y = \frac{\beta \mu}{1 + q + \beta \mu} \lambda_F \); in general equilibrium, this is equivalent to \( \lambda_F = \delta \).

**Directed search.** Following Moen (1997), a growing strand of literature has focused on market mechanisms allowing to decentralize the efficient allocation in matching models. The main market mechanism is the following: firms post wages (and doing so, precommit to pay the posted wage), and workers can direct their search towards a particular job. The papers by Acemoglu and Shimer (1999), and Shi (2001, 2002) show that this mechanism solves the hold-up problem that arises when firms make ex-ante investments. The key point in Acemoglu-Shimer’s and Shi’s papers is that firms both propose jobs and make ex-ante investments. As shown by Shi (2001), the wage posting/directed search mechanism achieves efficiency by properly allocating the decision rights on surplus division to agents who actively organize the market. However, in our paper, both firms and workers are market-organizers: firms advertise job offers, workers invest in education. By giving the

\[^{21}\text{The dependence of } W^u \text{ vis-à-vis } \theta \text{ has been made explicit.}\]
wage determination to only one side of the market, the wage posting/directed search mechanism fails to meet Shi’s requirement. As an illustration, consider the case where education only affects productivity, and assume each firm posts a wage along with a vacancy, while workers queue for jobs and matching is random. The unique equilibrium of the wage-posting game with endogenous education is the autarchic equilibrium where no firm enters the search market and workers do not educate. The reason is that the equilibrium posted wage, say \( w \), does not depend on education, and under free entry firms cannot commit to a credible skill requirement \( y(T) \) but the posted wage. As a result, workers choose the schooling duration that makes them as productive as the wage they will be paid, that is \( y(T) = w \). The corresponding flow profit is nil, and, therefore, the equilibrium posted wage \( w = 0 \) and no vacancy is advertised\(^{22}\).

Implementing the first best. As both the decentralized determination of tightness and that of education convey externalities, two instruments are required to compensate these two distortions. We now show the social optimum can be decentralized by means of appropriate tax/subsidy schemes on output and schooling duration.

Let \( \sigma \) be a voucher (tuition fee) per unit of time spent in the education system\(^{23}\). Let also \( \tau \) be a tax (subsidy) rate on value added \( y \). Free entry implies

\[
\frac{\gamma}{\eta} = \frac{(1 - \beta)(1 - \tau)y}{\delta + q + \beta Fm} \quad (28)
\]

A given individual aims at maximizing:

\[
\max_{T \geq 0} \left( e^{-\delta T} W^u(T) + \int_0^T e^{-\delta \tau} \sigma d\tau \right) \quad (29)
\]

The first-order condition reads as

\[
\lambda_y + \frac{\delta + q}{\delta + q + \beta \mu} \lambda_F = \delta - \frac{\delta + q + \beta \mu}{\beta \mu (1 - \tau)} \sigma \quad (30)
\]

**Proposition 6 First best in the decentralized economy**

The social optimum is decentralized if

\[
(i) \quad \tau = \tau^* = \frac{1 - \beta - \alpha}{1 - \beta} \frac{\delta + q + \mu}{\delta + q + \beta \mu^2},
\]

\[
(ii) \quad \sigma = \sigma^* = \frac{\sigma^*}{(1 - \tau^*) \eta^*} = \frac{\beta \mu^2 (\delta + q)}{1 - \beta (\delta + q + (1 - \alpha) \mu^2)} \left\{ (\delta - \lambda_F^*) + \lambda_F^* \left( 1 - \frac{\alpha^*}{(1 - \beta)(1 - \tau^*)} \right) \right\}
\]

\(^{22}\)If matching is not random and firms can observe education, workers are ranked in a job queue and we are back to Moen’s (1999) predictions. If firms post a wage bargaining rule, i.e. a \( \beta \in [0, 1] \), then the equilibrium is \( \beta = 1 - \alpha \), the Hosios condition. This, again, leads to undereducation. We may of course build more complex wage contracts relating the wage to human capital \( y \). This is beyond the scope of this paper.

\(^{23}\)We do not address the question of how to finance the voucher, or how to finance the fee. In the former case, the policy can be financed by mean of a lump-sum tax on all individuals. In the latter case, the fee can be given to newborn individuals.
The optimal labour market policy only responds to labour market distortions. When the Hosios condition holds, i.e. $\beta = 1 - \alpha^s$, the tax is nil. When $\beta < 1 - \alpha^s$, the economy spends too many resources in search spendings. The optimal policy, therefore, is to tax output and redistribute the proceeds of taxation to households. Conversely, when $\beta > 1 - \alpha^s$, tightness is too low, and the optimal policy is to set a tax on households to finance a subsidy to job creation.

The optimal education policy depends both on the working of the labour market and on the distortions involved by education. Hence, the sign of $\sigma^p$ depends on the sign of the sum of two terms into brackets. The first term corrects the distortions specific to schooling investments. It collapses when $\lambda_F^T = \delta$. The second term tackles with the labour market distortions involved by wage bargaining, which collapse when $\beta = 1 - \alpha^s$.

5 Unsegmented market

In this section, we assume workers apply to all jobs irrespective of the embodied technology. This assumption means there is a unique search market for all technologies, i.e. the market is unsegmented. One may imagine ads for jobs do not convey enough information on the type of skills required to perform the job, i.e. workers may not direct their search. This specification of the search process allows us to highlight the wage and congestion externalities underlying excess schooling.

The main change is that education does no longer affect the number of contacts between firms and vacancies, but the probability such contacts give rise to an employment relationship. Hence, the aggregate number of contacts only depends on the numbers of unemployed and vacancies, according to $M \equiv M(U, v)$. Tightness is then $x = v/U$. The flow probability for an individual with a duration of studies $T_i$ of becoming employed is still $\lambda_y^T = F(T_i) m(x)$. However, this is now the product of a contact rate $m(x)$ times the probability the embodied technology can be operated by the worker. More important, the flow probability for a firm to hire an applicant is $\eta(x) F(T)$, instead of $\eta(\theta)$. This is the rate of contact $\eta$ times the probability the worker can operate the technology. The asymmetry in the formulation of firms’ and workers’ matching rates prevailing in the segmented case does no longer apply in the unsegmented case. Raising education will translate into both shorter unemployment spells and shorter delays to fill vacancies, as it reduces the extent of the mismatch between firms’ technological requirements and workers’ skills.

The rest of the model remains unaltered. The arbitrage equations (4), (5), (6), (7) still hold with $\eta \equiv \eta(x) F(T)$. Hence the outcome of the wage bargain and the expected utility of an unemployed remain unchanged.

The optimal schooling duration solves:

$$\lambda_y^T \left( \bar{T} \right) + \lambda_F^T \left( \bar{T} \right) \frac{r + q}{r + q + \beta F(\bar{T}) m(x)} = r$$

This equation is identical to (13), save the relevant tightness $x$, which does not depend
on average schooling duration. Thus, comparative static properties remain the same. In particular, optimal schooling duration goes up as the incidence and duration of unemployment increase.

Free entry implies
\[
\frac{\gamma}{F(T) \eta(x)} = \frac{(1 - \beta) y(T)}{r + q + \beta F(T) m(x)}
\]

(32)

It is easy to check equation (32) defines tightness as an increasing function of the schooling duration, even if education does not alter productivity. This departs from the segmented case in an essential way. Indeed, the result of the asymmetry between firms and workers in the previous case was a negative wage externality, detrimental to job creation. When the market is unsegmented, the wage externality still takes place, but is actually dominated by the rise in the probability to fill a vacancy.

As equation (31) defines a decreasing relationship between education and tightness, while equation (32) defines an increasing relationship, there is a unique equilibrium. As in the segmented case, the equilibrium schooling duration is minimized whenever \(\beta = 1 - \alpha\).

Efficiency. The social planner is submitted to the same information restrictions as workers. Therefore, it is taken as given that workers search on all sub-markets, irrespective of whether they are able to operate the underlying technology or not. When \(\rho \to 0\), efficient schooling duration and tightness solve
\[
\lambda^s_x + \frac{\delta + q}{\delta + q + (1 - \alpha^s) F(T^*) m(x^*)} \lambda^p_F = \delta \left(1 - \frac{\alpha^s (\delta + q)}{\delta + q + (1 - \alpha^s) F(T^*) m(x^*)}\right)
\]

(33)

\[
\frac{\gamma}{\eta(x^*)} = \frac{\alpha^s}{\delta + q + (1 - \alpha^s) F(T^*) m(x^*)} F(T^*) y(T^*)
\]

(34)

Equations (33) and (34) must be compared to equations (31) and (32). The first point to make is that the decentralized equilibrium is always inefficient. When the Hosios condition holds the LHS of equations (31) and (33) are equal, but the RHS of equation (33) is lower than the RHS of equation (31) at given tightness and schooling duration. Consequently, and this is our second point, workers under-invest in education when the Hosios condition is satisfied. When the search market is unsegmented, raising one’s schooling investment does not create additional congestion effects for the other workers, as all job-seekers already search on all sub-markets. Instead, it reduces the extent of the mismatch by raising both workers’ and firms’ probability to meet an adequate partner. The result is that tightness is increasing in education, which is not internalized by students. Formally,
\[
MR^s = MR^p + \frac{\partial W^u}{\partial \theta} \frac{d\theta}{dT} > MR^p
\]

(35)

Note this result holds provided the Hosios condition is met. When the bargaining power \(\beta \to 0\), or \(\beta \to 1\), over-education may also take place.

Segmented vs unsegmented market. From a theoretical viewpoint, a fully segmented search market works more efficiently than an unsegmented one. To see this, compare
the flow number of matches in each case at given stock of unemployed $U$, vacancies $v$ and schooling duration $T$. In the segmented case, this is $M (UF (T), v)$, while in the unsegmented case, this is $F (T) M (U, v) < M (UF (T), v)$, because $M$ is homogenous of degree 1, and $F (T) < 1$. In the segmented case, all contacts lead to matches. A worker who embodies the skills needed for a certain type of job can be properly allocated faster if markets are segmented since only those with the adequate skills search in this case. In the unsegmented version, it takes more time to properly match the workers: a fraction of matches is unfruitful, and those workers cause congestion to those who embody the proper skills.

6 Conclusion

This paper is aimed at studying the incentives to invest in human capital when education is a time-consuming activity and determines jointly the scope – or adaptability – and intensity – or productivity – of individual skills. We build a search equilibrium model with ex-post wage bargaining where workers have multidimensional skills and the search market is segmented by technology. We establish three main results. First, unemployment provides incentives to schooling by raising the need for adaptability. Second, private returns are below social returns when education determines only the productivity but no hold-up phenomenon is involved. Rather, this is the result of a participation externality: an increase in schooling duration reduces the steady-state number of unemployed, and thus lowers search costs at given labour market tightness. Third, the adaptability component of education creates novel wage and congestion externalities and the private returns to schooling may be higher than the social returns. As a consequence, both over- and under-education may take place in equilibrium.
APPENDIX

1 Proofs

Proof of proposition 1 (i) Let $G(T) = e^{-rT} \frac{\beta F(T)m}{r+q+\beta F(T)m}$. Since $G(0) = G(\infty) = 0$ and $G$ is differentiable over $[0, \infty)$, the first order condition is necessary. It yields equation (13). Consider function $\phi_1$ such that

$$\phi_1(T, \theta) = \lambda_y(T) + \lambda_F(T) \frac{r+q}{r+q+\beta F(T)m(\theta)} - r \quad (36)$$

From Assumption 1, $\phi_1$ is continuously differentiable in $T$ with

$$\lim_{T \to 0} \phi_1 = +\infty \quad \text{and} \quad \lim_{T \to +\infty} \phi_1 = -r \quad (37)$$

Taking its derivative with respect to $T$,

$$\frac{\partial \phi_1}{\partial T} = \lambda_y' + \frac{r+q}{r+q+\beta \mu} \lambda_F' - \frac{r+q}{r+q+\beta \mu} \frac{\beta \mu}{r+q+\beta \mu} \lambda_F^2 < 0 \quad (38)$$

because $y$ and $F$ are strictly concave by Assumption 1. Therefore, there exists a unique $T = \hat{T}(\theta)$ such that $\phi_1\left(\hat{T}, \theta\right) = 0$ to which the implicit function theorem applies.

(ii) For $T = \hat{T}$, we have

$$\frac{\partial \phi_1}{\partial \theta} = \frac{\alpha}{\theta} \frac{\beta \mu}{r+q+\beta \mu} (\lambda_y - r) < 0 \quad (39)$$

$$\frac{\partial \phi_1}{\partial \beta} = \frac{\mu}{r+q+\beta \mu} (\lambda_y - r) < 0 \quad (40)$$

$$\frac{\partial \phi_1}{\partial q} = \frac{\beta \mu}{r+q+\beta \mu} r - \lambda_y > 0 \quad (41)$$

$$\frac{\partial \phi_1}{\partial r} = \frac{\beta \mu}{r+q+\beta \mu} r - \lambda_y - 1 < 0 \quad (42)$$

It follows that $d\hat{T}/d\theta < 0$, $d\hat{T}/d\beta < 0$, $d\hat{T}/dq > 0$, and $d\hat{T}/dr < 0$.

Proof of proposition 2 (i) Let function $\phi_2$ be such that

$$\phi_2(T, \theta) = \frac{\gamma}{\eta(\theta)} - \frac{(1-\beta) y(T)}{r+q+\beta F(T)m(\theta)} \quad (43)$$

From Assumptions 1 and 2, $\phi_2$ is continuously differentiable in $\theta$, with

$$\lim_{\theta \to 0} \phi_2 = -(1-\beta) y/(r+q) \quad \text{and} \quad \lim_{\theta \to +\infty} \phi_2 = +\infty \quad (44)$$
Taking its derivative with respect to \( \theta \) yields

\[
\frac{\partial \phi_2}{\partial \theta} = \frac{1 - \alpha}{\eta} \gamma + \frac{\alpha (1 - \beta)}{\eta r + q + \beta \mu} r + q + \beta \mu > 0
\]  (45)

where \( \alpha \equiv \alpha (\theta) \equiv \theta m'(\theta) / m(\theta) \in (0, 1) \) due to Assumption 2. Consequently, there exists a unique \( \hat{\theta} \equiv \hat{\theta} (T) \) such that \( \phi_2 (T, \hat{\theta}) = 0 \) to which the implicit function theorem applies.

(ii) and (iii). For \( \theta = \hat{\theta} \), we have

\[
\begin{align*}
\frac{\partial \hat{\theta}}{\partial T} & \equiv \frac{\gamma}{\eta} \left( -\lambda_y + \frac{\beta \mu}{r + q + \beta \mu} \right) > 0 \quad (46) \\
\frac{\partial \hat{\theta}}{\partial \beta} & \equiv \frac{\gamma}{\eta r + q + \beta \mu} > 0 \quad (47) \\
\frac{\partial \hat{\theta}}{\partial q} & \equiv \frac{\gamma}{\eta r + q + \beta \mu} > 0 \quad (48) \\
\frac{\partial \hat{\theta}}{\partial r} & \equiv \frac{\gamma}{\eta r + q + \beta \mu} > 0 \quad (49)
\end{align*}
\]

It follows that \( \frac{d\hat{\theta}}{d\beta} < 0, \frac{d\hat{\theta}}{dq} < 0, \frac{d\hat{\theta}}{dr} < 0, \) and \( \frac{d\hat{\theta}}{dT} \) has the sign of \( \lambda_y - \frac{\beta \mu}{r + q + \beta \mu} \).

**Proof of proposition 3** An equilibrium is a pair \((\theta^*, T^*)\) which solves \( \phi_1 (T^*, \theta^*) = 0, \phi_2 (T^*, \theta^*) = 0 \), or, equivalently, \( T^* = \hat{T} (\theta^*) \) and \( \theta^* = \hat{\theta} (T^*) \).

(i) We proceed in two steps. First, we show there exists an equilibrium. Second, we prove it is unique.

**Step 1. Existence.** From proposition 1, \( \hat{T} \) is continuously and strictly decreasing on \([0, \infty)\). Moreover, \( \lim_{\theta \to 0} \hat{T} = (\lambda_y + \lambda_F)^{-1} (r) \equiv T_{\max} \) and \( \lim_{\theta \to +\infty} \hat{T} = \lambda_y^{-1} (r) \equiv T_{\min} \)  (50)

We can therefore define \( \overline{T} \equiv \hat{T}^{-1} \), which is continuously differentiable and strictly decreasing on \([T_{\min}, T_{\max}]\). Moreover,

\[
\begin{align*}
\lim_{T \to T_{\min}} \overline{T} = 0 \quad \text{and} \quad \lim_{T \to T_{\max}} \overline{T} = +\infty
\end{align*}
\]  (51)

From proposition 2, \( \hat{\theta} \) is continuously differentiable on \([0, \infty)\), with

\[
\begin{align*}
\lim_{T \to 0} \hat{\theta} = 0 \quad \text{and} \quad \lim_{T \to T_{\max}} \hat{\theta} = \theta_{\max} > 0
\end{align*}
\]  (52)

where \( \theta_{\max} \) is finite or infinite according to \( \lim_{T \to +\infty} y (T) \) is finite or not. By continuity, there exists \( T^* \) which solves \( \hat{\theta} (T^*) = \overline{T} (\theta^*) = \theta^* \).
Step 2. *Uniqueness.* Let \( J \) denote the Jacobian matrix of function \( \Phi \equiv (\phi_1, \phi_2) \) evaluated in equilibrium.

\[
J = 
\begin{bmatrix}
\frac{\partial \phi_1^*}{\partial T} & \frac{\partial \phi_1^*}{\partial \theta} \\
\frac{\partial \phi_2^*}{\partial T} & \frac{\partial \phi_2^*}{\partial \theta}
\end{bmatrix}
\]  
(53)

where partial derivatives are:

\[
\frac{\partial \phi_1^*}{\partial T} = \lambda_t' + \lambda_F \frac{r + q}{r + q + \beta \mu} + (\lambda_t - r) (\lambda_y + \lambda_F - r)
\]  
(54)

\[
\frac{\partial \phi_1^*}{\partial \theta} = \frac{\beta \mu}{\theta} \frac{r + q + \beta \mu}{r + \beta \mu} (\lambda_t - r)
\]  
(55)

\[
\frac{\partial \phi_2^*}{\partial T} = \frac{\gamma}{\eta} (\lambda_F - r)
\]  
(56)

\[
\frac{\partial \phi_2^*}{\partial \theta} = \frac{1}{\theta} \frac{\gamma \beta \mu + (1 - \alpha)(r + q)}{r + q + \beta \mu}
\]  
(57)

The determinant of matrix \( J \) is

\[
\det J = \frac{\gamma}{\eta r + q + \beta \mu} \left[ \lambda_t' + \lambda_F \frac{r + q}{r + q + \beta \mu} + (\lambda_t - r) (\lambda_y + \lambda_F - r) \right] - \frac{\beta \mu}{\eta (r + q + \beta \mu)} (\lambda_t - r)
\]  
(58)

As \( \lambda_t' < 0, \lambda_F < 0, \lambda_t < r, \) and \( \lambda_y + \lambda_F > r, \)

\[
\det J < \frac{\gamma}{\eta r + q + \beta \mu} (\lambda_t - r) [(1 - \alpha)(\lambda_y + \lambda_F - r) + \alpha \lambda_t] < 0
\]  
(59)

Uniqueness follows.

(ii) and (iii). *Comparative statics.* From (i), there exists a unique equilibrium. The implicit function theorem then implies that for any parameter \( p, \)

\[
\begin{bmatrix}
\frac{dT^*}{dp} \\
\frac{d\theta^*}{dp}
\end{bmatrix} = -J^{-1} \begin{bmatrix}
\frac{\partial \phi_1^*}{\partial p} \\
\frac{\partial \phi_2^*}{\partial p}
\end{bmatrix}
\]  
(60)

where

\[
\frac{\partial \phi_1^*}{\partial \beta} = \frac{\mu}{r + q + \beta \mu} (\lambda_y - r)
\]  
(61)

\[
\frac{\partial \phi_1^*}{\partial q} = \frac{\beta \mu}{r + q + \beta \mu} \frac{r - \lambda_y}{r + q}
\]  
(62)

\[
\frac{\partial \phi_2^*}{\partial \beta} = \frac{(1 - \beta) \mu y}{r + q + \beta \mu r + q + \beta \mu}
\]  
(63)

\[
\frac{\partial \phi_2^*}{\partial q} = \frac{(1 - \beta) \mu y}{r + q + \beta \mu r + q + \beta \mu}
\]  
(64)

and

\[
J^{-1} = \frac{1}{\det J} \begin{bmatrix}
\frac{\partial \phi_2^*}{\partial T} & -\frac{\partial \phi_1^*}{\partial T} \\
-\frac{\partial \phi_2^*}{\partial \theta} & \frac{\partial \phi_1^*}{\partial \theta}
\end{bmatrix}
\]  
(65)
Proof of proposition 4 The planner’s objective can be written as

$$C = C (T, \theta) \equiv e^{-\delta T} (1 - u) [y - (q + \delta) \gamma / \eta]$$

(66)

We first prove first-order conditions are necessary. As $C (0, \theta) = -(q + \delta) \gamma / \eta (\theta) < 0$, $C (T, 0) = 0$, $C (\infty, \theta) = 0$, $C (T, \infty) = -\infty$, and $C$ is continuously differentiable, it is sufficient to show $C (T, \theta) > 0$ for some $T > 0$ and some $\theta > 0$. This is straightforward as $y (T) - (q + \delta) \gamma / \eta (\theta)$ varies from $y (T)$ to minus infinity when $\theta$ varies from 0 to infinity.

Manipulating the FOC and using the fact that $-\theta \eta' (\theta) / \eta (\theta) = 1 - \theta m' (\theta) / m (\theta) = 1 - \alpha (\theta)$ yield equations (16) and (17). ■

Proof of proposition 5 We know from proposition 3 the equilibrium schooling duration $T^*$ reaches a minimum when $\beta = 1 - \alpha^*$. To show points (i) and (ii), it is therefore sufficient to prove $T^* (1 - \alpha^*) > T^*$. To this aim, consider function $\psi$ such that

$$\psi (T, \theta) = (1 - \alpha^*) \frac{\delta + q}{\delta + q + (1 - \alpha^*) F (T) m (\theta)} (\lambda_F (T) - \delta)$$

(67)

Efficient schooling and tightness result from

$$\phi_1 (T, \theta, 1 - \alpha^*) - \psi (T, \theta) = 0$$

(68)

$$\phi_2 (T, \theta, 1 - \alpha^*) = 0$$

(69)

while $T^* (1 - \alpha^*)$ is defined by

$$\phi_1 (T, \theta, 1 - \alpha^*) = 0$$

(70)

$$\phi_2 (T, \theta, 1 - \alpha^*) = 0$$

(71)

Let $\overline{T} \equiv \overline{T} (\theta)$ denote the solution of equation (68), while $\hat{T} \equiv \hat{T} (\theta)$ defined in proposition 1 denote the solution of equation (70).

If $\lambda_F^* > \delta$, then $\psi (\cdot) > 0$. As $\phi_1$ is strictly increasing in $T$, this implies $\hat{T} (\theta) > \overline{T} (\theta)$ for all $\theta > 0$. It follows $T^* < T^* (1 - \alpha^*) \leq T^* (\beta)$ for all $\beta \in (0, 1)$, i.e. (i) is proved.

If $\lambda_F^* = \delta$, then $\psi (\cdot) = 0$, and $T^* = T^* (1 - \alpha^*) < T^* (\beta)$ for all $\beta \neq 1 - \alpha^*$, i.e. (ii) is proved.

Finally, if $\lambda_F^* > \delta$, $\psi (\cdot) < 0$. This implies $\hat{T} (\theta) < \overline{T} (\theta)$ for all $\theta > 0$. Therefore, $T^* > T^* (1 - \alpha^*)$. By continuity there exist $\beta_-$ and $\beta_+$ such as defined in (iii). ■

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24 The third argument in functions $\phi_1$ and $\phi_2$ highlights the dependency vis-à-vis $\beta$. 

25
Proof of proposition 6 The result is obtained by identifying equation (17) to equation (28), and equation (16) to (30). More precisely, (17) and (28) imply that

\[
\frac{(1 - \beta)(1 - \tau^p)}{\delta + q + \beta \mu^s} = \frac{\alpha^s}{\delta + q + (1 - \alpha^s) \mu^s}
\]  

(72)

Point (i) follows. Then, using (16) and (30), we have

\[
\lambda_F^r \left\{ \frac{\delta + q}{\delta + q + \beta \mu^s} - \frac{(1 - \alpha^s)(\delta + q)}{\delta + q + (1 - \alpha^s) \mu^s} \right\} = \frac{\delta + q + \beta \mu^s}{\beta \mu^s} \frac{\alpha^s}{(1 - \tau^s) \mu^s} + \frac{\alpha^s(\delta + q) \delta}{\delta + q + (1 - \alpha^s) \mu^s}
\]

(73)

Together with (72), we get (ii).
References


Fig. 1a: Schooling and bargaining power
Case $\lambda_F > \delta$

Fig. 1b: Schooling and bargaining power
Case $\lambda_F < \delta$