Self-selection in education with matching frictions*

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Abstract: This paper studies the efficiency of educational choices in a two sector/two schooling level matching model of the labour market where a continuum of heterogeneous workers allocates itself between sectors depending on their decision to invest in education. We show self-selection in education is inefficient, and overeducation takes place. Too many workers are willing to acquire education, as they do not internalize the impact of their education decision on the others’ wage and employment perspectives.

Keywords: Matching frictions; Education; Heterogeneity; Efficiency

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1. Introduction

In the presence of matching unemployment, it is a well established result that the incentives to education are generally too weak. On the one hand, it has been emphasized that workers’ decision to invest in education and firms’ job creation decisions interact positively, so that the economy can get stuck in a “bad” equilibrium with both insufficient human capital investment and high unemployment (see e.g. Laing et al 1995, or Burdett and Smith, 2002). On the other hand, frictional unemployment can create a hold-up problem (Acemoglu, 1996): search frictions give rise to rent sharing, which implies workers are generally paid a share less than one of their marginal productivity. Underinvestment will result when workers bear alone the full cost of their education.

In this paper, we show that, contrary to common belief, search costs/unemployment can induce overeducation. Our argument is based on market segmentation, workers’ heterogeneity and self-selection in education. When educational attainment and labour market ability are positively correlated, self-selection gives rise to a composition effect, according to which the mean ability among each education group decreases when the share of educated rises. As far as the labour demand responds to workforce productivity, this composition effect translates into lower employment opportunities for both education groups. Individuals do not take this effect into account while self-selecting in education. As a consequence, too many workers are willing to educate, i.e. overeducation takes place.

We analyse the steady-state(s) of a two-sector matching model where workers are heterogenous and free entry of firms drives job creation. Workers can direct their search between two perfectly segmented labour markets, one for the educated and one for the uneducated, as in Saint-Paul (1996). Workers’ productivity can be split in two parts: the first part is firm or sector-specific and can be related to education. The other component is innate to the worker and, for lack of a better term, we call it ability. Education is costly, and the cost is assumed to be the same for all workers, i.e. ability-independent. Heterogeneity of the labour force and schooling costs give rise to self-selection in the educational system: as workers are charged a fixed cost for their schooling investment (e.g. tuition fees), only those with a sufficiently high ability choose to invest in education and are qualified to work on high productivity jobs.

Self-selection is inefficient: too many workers choose to invest in education. This results from a composition effect. As only the ablest get an education, the number of educated rises only if less able individuals are drawn into education: the ablest among the former uneducated now become the least able among the educated. As a result, the
average productivity across the two education groups falls.

In a frictionless world, this composition effect would not affect individual returns to schooling. The earnings distribution would change, but the equilibrium would be socially efficient. With matching frictions, this composition effect has dramatic implications for efficiency. Self-selection creates an important externality: the composition of education groups affects firms’ incentives to enter each sector. A rise in the number of educated workers deteriorates search prospects for firms in both sectors, which in turn advertise fewer vacancies. This gives rise to overeducation: the threshold individual considers her own earnings and employment perspectives, which improve by schooling, but does not internalize the impact of her schooling decision on the others’ job opportunities. This leads to a welfare loss. The optimal education policy is thus to set a tax on education to deter low-ability individuals from participating to the high-productivity sector.

Several studies have emphasized the importance of composition effects arising from self-selection in education, building on Roy’s (1951) multi-sector model of occupational choices. For instance, Willis and Rosen (1979), Cameron and Heckman (1998) study the impact of self-selection on the returns to schooling. According to Cameron and Heckman, p. 316, “analyses that ignore heterogeneity in student ability may present an overly optimistic account of the likely effect of policies that promote college attendance on the wages of college graduates as a whole”. Heckman et al (1998a, 1998b) build dynamic general equilibrium models with education choices where composition effects explain part of the observed changes in the distribution of earnings and returns to schooling in the US over the past decades. Our paper exploits the basic insights of these models, i.e. that self-selection at the individual level alters the composition of education groups at the aggregate level. Our contribution to this literature is then to highlight a novel externality arising from self-selection when search frictions and firms’ entry are an integral part of the trading environment\(^1\). The paper by Andolfatto and Smith (2001) is closest to us. It incorporates search-type frictions in a Roy model, and focuses on the dynamics of sectoral employment following biased productivity shocks. A key difference is that employment opportunities do not respond to the composition of education groups in their paper\(^2\).

Some studies have already elaborated on the idea that there can be overeducation in

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\(^1\)The externality highlighted here vanishes as search frictions die down; the model then collapses to the fully competitive version of Roy’s model where workers’ ability is perfectly correlated across sectors.

\(^2\)It is assumed workers may undertake a (re)training period following a sector-specific productivity shock. At the end of the retraining period, there is an \emph{exogenous} per period probability that training is completed.
frictional environments. Saint-Paul (1996) builds a dual model with a fixed number of jobs, where education decisions alter firms’ incentives to allocate their vacancies between sectors. In Moen (1999), workers queue for jobs. Education improves a worker’s chance of getting a job, but reduces that of the others. In Charlot et al (forthcoming), the search market is segmented by technology and schooling improves the number of technologies a worker can operate. Education not only improves the exit rate from unemployment, but can also be used to raise one’s outside opportunities during the wage bargain. The latter effect is not met by a social gain. We add a novel externality to this literature that is derived from composition effects associated to self-selection in the schooling system.

Our model is related to other matching models of assignment, e.g. Sattinger (1995), Burdett and Coles (1997), Burdett and Wright (1998), or Mortensen and Wright (2002). In these papers, the search market is not segmented, and the assignment of workers to jobs results from workers’ and firms’ strategies of setting a reservation productivity. The existence of those non-trivial reservation productivities stems from the assumption that a fixed number of workers meets a fixed number of firms on a single market with two-sided heterogeneity. In our model, the assignment of workers to sectors is driven by self-selection in education and free entry drives the value of the reservation productivity to zero. In equilibrium, it is thus never optimal to reject a match to wait for a better opportunity. No decentralized mechanism is at play to deter the participation of low ability individuals to the high-skill sector. In the absence of collective agreements to set minimum standards in the market for skills, there is too little self-selection. A possible alternative to such market failures is therefore a more stringent education policy.

We emphasize overeducation as a potential outcome of private educational choices, a theoretical prediction that is also shared by signalling models of the labour market (e.g. Arrow, 1973, Spence, 1973). Signalling models rely on two main assumptions: unobserved heterogeneity of the workforce and costs of schooling. Education is a socially wasteful activity to the extent that it consumes more resources than it raises productivity. In this paper, heterogeneity is crucial for the results, but it is actually observed by firms when a match occurs. The main difference is that overeducation has nothing to do with the amount of resources devoted to education, since this situation also occurs when education is free (this point is formally demonstrated in section 4).

The outline of the paper is as follows: Section 2 describes the theoretical framework. Section 3 details the decentralized allocation. Section 4 discusses social efficiency and normative aspects of education policy. Section 5 offers a discussion of our results. Section
6 concludes.

2. The model

We describe the steady state of a two sector/two schooling level matching model of unemployment with heterogenous workers and endogenous schooling choices. Time is continuous. At each instant, $\delta > 0$ agents are born into unemployment. Each individual is facing a constant risk of dying $\delta$, so that the population is constant (and normalized to unity). Individuals in each cohort differ in a single characteristic $a$ distributed across each cohort according to the cumulative distribution function $\Phi$ on $[0, 1]$, and associated density function $\phi > 0$ and continuous over the unit interval. As a starting point, we assume that a fixed proportion of the population becomes highly educated, while the rest remain low educated. The proportions will be endogenized later. Individuals are risk neutral. Let $\rho$ denote the individual discount rate as well as the interest rate of the economy. The economy is made up of two sectors producing a single good. Variables are indexed by $i$, where $i = h$ stands for high, or $i = l$ for low. In each sector there is a large number of firms, each endowed with a single vacancy. The output $y_i$ of a match depends on the worker’s characteristic $a$ and on the sector-specific productivity parameter $A_i$: $y_i(a) = A_i a$. Let $A_h > A_l > 0$. The technology currently in use on a high skill occupation is more efficient than that used on a low skill occupation.

2.1. The matching sector

Vacant jobs and unemployed workers are brought together in pairs through an imperfect matching process. Let $u_i$ and $v_i$ denote respectively the numbers of unemployed and vacancy on market $i$. The total number of employer-worker contacts $M_i$ on market $i$ is given by the following matching technology:

$$M_i = M(u_i, v_i)$$

The function $M : [0, +\infty) \times [0, +\infty) \to [0, +\infty)$ is twice continuously differentiable, strictly increasing in each of its arguments, strictly concave and linearly homogenous. It satisfies the boundary conditions $M(u_i, 0) = M(0, v_i) = 0$, and the Inada conditions:

$$\lim_{u_i \to 0} \partial M/\partial u_i = \lim_{v_i \to 0} \partial M/\partial v_i = \infty, \text{ and } \lim_{u_i \to \infty} \partial M/\partial u_i = \lim_{v_i \to \infty} \partial M/\partial v_i = 0.$$ 

Matches are equiprobably distributed between the unemployed and the vacancies; linear homogeneity of the matching function allows us to express the flow probabilities for a vacant job (unemployed worker) to meet an unemployed (vacancy) as a function of the
labour market tightness $\theta_i \equiv u_i / u$. Let $m(\theta_i) \equiv M(1, \theta_i)$. The unemployed meet jobs on market $i$ at rate $M_i / u_i \equiv m(\theta_i)$, an increasing function of $\theta_i$. Similarly, the rate at which vacancies are filled on market $i$ is $M_i / v_i \equiv m(\theta_i) / \theta_i$, a decreasing function of $\theta_i$.

2.2. Firms’ and workers’ gains

Let $r \equiv \rho + \delta$ be the effective discount rate. Let $U_i(a)$ and $W_i(a)$ be the asset values of being unemployed and employed in sector $i$ for a worker with type $a$. Assuming (for simplicity) no job quits and no unemployment benefits, we have:

$$rU_i(a) = m(\theta_i) [W_i(a) - U_i(a)] \quad (2.2)$$
$$rW_i(a) = w_i(a) \quad (2.3)$$

where $w_i(a)$ denotes the wage of a worker endowed with ability $a$. Job seekers move into employment with probability $m(\theta_i)$ and enjoy a gain from change of state equal to $W_i(a) - U_i(a)$. Symmetrically, let $V_i$ and $J_i(a)$ denote the asset values of holding a vacancy and a filled job in sector $i$. Those values satisfy the following two arbitrage equations:

$$\rho V_i = -\gamma + (m(\theta_i) / \theta_i) [E(J_i(a)) - V_i] \quad (2.4)$$
$$rJ_i(a) = y_i(a) - w_i(a) + \delta V_i \quad (2.5)$$

Holding a vacancy induce a flow cost $\gamma > 0$. Vacancies may become filled with probability $m(\theta_i) / \theta_i$. Filled jobs bring a revenue to the firm equal to $y_i(a) - w_i(a)$. Wages are determined by Nash bargaining over the match surplus $y_i / r - U_i - V_i$. This implies:

$$\beta [J_i(a) - V_i] = (1 - \beta) [W_i(a) - U_i(a)] \quad (2.6)$$

where $\beta \in (0, 1)$ is the (exogenous) bargaining power of the worker. Free entry drives the value of a vacancy down to zero. For each sector $i$, the expected value of a filled job is therefore equal to the expected search cost:

$$E[J_i(a)] = \gamma \theta_i / m(\theta_i) \quad (2.7)$$

2.3. The schooling sector

We assume schooling is costly. Let $C > 0$ denote the (ability-independent) schooling cost. Let also $S$ denote the level of the schooling subsidy granted by the State\(^3\). Schooling

\(^3\)When the subsidy is positive, it is implicitly financed by a (non-distorsionary) lump-sum tax on all workers, whatever their education level or employment status. However, the optimal subsidy is typically
costs imply that heterogenous workers select themselves through the education system. At birth, each person compares her lifetime utility if educated (net of schooling costs) to that she would get if uneducated and invests accordingly. An individual endowed with characteristic $a$ decides to get an education if and only if

$$U_h(a) \geq U_l(a) + C - S$$

(2.8)

The RHS is the sum of the opportunity cost $U_l(a)$, plus the direct schooling cost $C$ minus the subsidy $S$. The utility levels $U_h(a)$ and $U_l(a)$ depend on expected wages but also on the job finding-rate $m(\theta_i)$ in each market. In the remainder, we shall denote by $\sigma \in (0, 1)$ the (endogenous) cut-off point below which individuals do not acquire education$^4$.

2.4. Flows and stocks

The size of the labour force $n_i$ on market $i$ evolves according to

$$dn_i/dt = \delta \sigma_i - \delta n_i$$

(2.9)

where $\sigma_i$ is the share of individuals entering the type $i$ population. By assumption, we have $\sigma_h = 1 - \Phi(\sigma)$ and $\sigma_l = \Phi(\sigma)$. The mass-numbers of educated and uneducated job seekers $u_i$ obey the following law of motion

$$du_i/dt = \delta \sigma_i - (m(\theta_i) + \delta) u_i$$

(2.10)

New entrants make up the inflow into unemployment. The outflow is equal to the sum of new hires and the number of deaths. Let $u_i^*$ denote the unemployment rate for education group $i$. It is related to the number of unemployed $u_i$ according to $u_i = u_i^* \sigma_i$.

2.5. Composition effect

The average abilities across the pools of unemployed in each sector are functions of the cut-off ability $\sigma$, and satisfy

$$E_l(\sigma) = \int_0^\sigma \frac{\phi(a)}{\Phi(\sigma)} ada \quad \text{and} \quad E_h(\sigma) = \int_\sigma^1 \frac{\phi(a)}{1 - \Phi(\sigma)} ada$$

(2.11)

negative as we shall see. The fee is redistributed within the entrants, or to the whole population by means of a lump-sum transfer.

$^4$We only focus on equilibrium configurations compatible with the existence of both markets.
Thus a shift in the selection threshold $\sigma$ involves a composition effect. By differentiating (2.11) w.r.t. $\sigma$ we get

$$\frac{dE_l(\sigma)}{d\sigma} = \frac{\phi(\sigma)}{\Phi(\sigma)} [E_l(\sigma) - \sigma] > 0$$

(2.12)

$$\frac{dE_h(\sigma)}{d\sigma} = \frac{\phi(\sigma)}{1 - \Phi(\sigma)} [\sigma - E_h(\sigma)] > 0$$

(2.13)

The average productivity in each education group is increasing in $\sigma$. When $\sigma$ goes up (i.e. the number of people in education goes down), the least able among the educated become the ablest among the uneducated. The mean ability in each education group increases. In the following two sections, we show how this composition effect interacts with firms’ incentives to enter each sector, and how an externality arises.

3. Equilibrium unemployment

In this section, we investigate the steady-state consequences of the composition effect arising from self-selection in education. We proceed in three steps. First, we determine wages, tightness and the selection threshold. Secondly, we give sufficient conditions for existence and uniqueness of equilibrium. Thirdly, we discuss the comparative statics of the model, and the possibility of multiple equilibria.

3.1. Closing the model

The wage bargain. Equations (2.2) to (2.6) determine the wage a worker with ability $a$ earns in sector $i$:

$$w_i(a) = \beta \frac{r + m(\theta_i)}{r + \beta m(\theta_i)} A_i a$$

(3.1)

Equation (3.1) is standard. The wage derived from the wage bargain rule (2.6) is equal to a share $\beta \frac{r + m(\theta_i)}{r + \beta m(\theta_i)}$ of the output flow $A_i a$ generated by a match. The wage rises with labour market tightness $\theta_i$, the output flow $A_i a$, and the bargaining power $\beta$. A rise in the output flow $A_i a$ raises the size of the surplus to be shared between a firm and a worker, and translates into a higher wage. A rise in the bargaining power $\beta$ raises the share of the surplus accruing to workers. Higher tightness implies that the workers’ fallback position $U_i$ improves at the bargaining stage because job offers arrive at a higher rate.

The free-entry condition. Using equations (2.5), (2.7) and (3.1), market tightnesses $\theta_h$ and $\theta_l$ are determined by the free-entry conditions:

$$\gamma \theta_i / m(\theta_i) = (1 - \beta) A_i E_i(\sigma) / (r + \beta m(\theta_i)) , \ i = h, l$$

(3.2)
$A_h > A_l$ and $E_h(\sigma) > E_l(\sigma)$ imply $\theta_h > \theta_l$. The job-finding rate $m(\theta_h)$ in the high-productivity sector is larger than the job-finding rate $m(\theta_l)$ in the low-productivity sector.

The self-selection rule. We now turn to the determination of the threshold. The threshold individual, with innate ability $\sigma$, is indifferent between education and no education. The cut-off ability $\sigma \in (0, 1)$, if it exists, is such that $U_h(\sigma) = U_l(\sigma) - (C - S)$. This gives:

$$\sigma = \frac{C - S}{\beta m(\theta_h) A_h r + \beta m(\theta_l) A_l r} \in (0, 1)$$

Equation (3.3) is a self-selection rule. At given tightness, the ability threshold $\sigma$ increases with schooling cost $C - S$, decreases with the utility gap between high education and low education. The utility gap depends on labour market tightness $\theta_h$ and $\theta_l$, as well as on the levels of sector-specific productivities $A_h$ and $A_l$. A rise in labour market tightness $\theta_h$ raises the incentive to become educated and causes a fall in the selection threshold $\sigma$. A fall in $\theta_l$ does the same. A rise in sector-specific productivity $A_h$ raises the reward associated with being educated and this causes a drop in the cut-off ability $\sigma$. A fall in $A_l$ does the same.

3.2. Existence and uniqueness of equilibrium

An interior equilibrium is a vector $(\theta^*_{ih}, \theta^*_{il}, \sigma^*)$ which solves the free entry conditions (3.2) for $i = h, l$ and the self-selection condition (3.3). We only consider equilibrium in which both sectors coexist.

The equilibrium can be solved recursively. Equation (3.2) defines a unique market tightness for labour market $i$, as a function of the cut-off ability $\sigma$, $\theta_i \equiv \Theta_i(\sigma)$. The job-finding rate in sector $i$ is $m(\Theta_i(\sigma))$. Solving (3.3) for $\sigma^* \in (0, 1)$ yields the proposition.

**Proposition 1** Let $\psi_i(\sigma) \equiv \frac{\beta m(\Theta_{ih}(\sigma)) A_i}{r + \beta m(\Theta_{ih}(\sigma)) A_i r}$ and $\varepsilon(\sigma) \equiv \frac{\sigma(\psi_h(\sigma) - \psi_l(\sigma))}{\psi_h(\sigma) - \psi_l(\sigma)}$. Then,

(i) There exists an interior equilibrium if

$$\lim_{\sigma \to 1} \{\psi_h(\sigma) - \psi_l(\sigma)\} > C - S > 0$$

(ii) The equilibrium is unique if, in addition,

$$\varepsilon(\sigma) > -1, \text{ for all } \sigma \in (0, 1)$$

The function $\psi_i$ is the return to ability in sector $i$. Therefore, the differential return to ability $\psi_h(\sigma) - \psi_l(\sigma)$ is also the return to schooling. An interior equilibrium is such
that the return to schooling times the threshold ability is equal to the schooling cost, that is \( [\psi_h (\sigma^*) - \psi_l (\sigma^*)] \sigma^* = C - S \) and \( \sigma^* \in (0,1) \). Existence and uniqueness of such an interior equilibrium are not guaranteed. Proposition 1 provides sufficient conditions for (i) existence and (ii) uniqueness of the equilibrium. We now develop the intuitions underlying those conditions.

(i) If the schooling cost is too high, the only equilibrium is degenerate with a single, low-productivity sector\(^5\). To rule out this case, the sufficient condition (3.4) states that the return to schooling should be larger than the schooling cost \( C_S \) as the threshold ability tends to its upper-limit (i.e. as \( \sigma \to 1 \)). This guarantees the ablest individual acquires education (and, by continuity, some less able individuals do so). As it turns out, the return to ability in each sector \( \psi_i \) is strictly increasing in sector-specific productivity \( A_i \), and is independent of the other sector-specific productivity, and of the schooling cost \( C - S \). Therefore, condition (3.4) is satisfied if \( A_h \) is sufficiently high and/or \( A_l \) is sufficiently low and/or \( C_S \) is sufficiently low.

(ii) The composition effect interacts with the firms incentives to enter each sector, and thus affects the return to schooling. Formally, the marginal impact of the threshold ability \( \sigma \) on the return to schooling is \( \psi'_h (\sigma) - \psi'_l (\sigma) \). The keypoint is that sector-specific returns to ability \( \psi_h \) and \( \psi_l \) are both increasing in \( \sigma \), which may prevent the uniqueness of equilibrium. To see this, differentiate \( \psi_i \) with respect to \( \sigma \):

\[
\psi'_i (\sigma) = \psi_i (\sigma) \frac{r \alpha_i}{r + \beta m (\Theta_i (\sigma))} \frac{1}{\Theta_i (\sigma)} \frac{d \Theta_i (\sigma)}{d \sigma} \tag{3.6}
\]

where \( \alpha_i \equiv \alpha (\Theta_i (\sigma)) \equiv \Theta_i (\sigma) m' (\Theta_i (\sigma)) / m (\Theta_i (\sigma)) \) is the elasticity of the matching function with respect to the number of vacancies. Threshold ability \( \sigma \) impacts the return to ability via its effect on sector-specific tightness \( \theta_i \). From the free-entry condition\(^6\) (3.2), we can compute the marginal impact of threshold ability on sector-specific tightness:

\[
\frac{d \Theta_i (\sigma)}{d \sigma} = \frac{r + \beta m (\Theta_i (\sigma))}{r (1 - \alpha_i) + \beta m (\Theta_i (\sigma))} \Theta_i (\sigma) \frac{E_i' (\sigma)}{E_i (\sigma)} > 0 \tag{3.7}
\]

Tightness is increasing in threshold ability. Mean ability, in each sector, is increasing in \( \sigma \). Due to rent-sharing, the higher the sector-specific mean ability, the larger the firms’ expected profits in the sector. The larger profits attract more firms in the industry (under free entry), tightness rises, the probability of finding a job increases.

\(^5\)In addition to such equilibria, note there always exist autarchic equilibria, where workers do not participate to the labour market, and, consequently, firms do not advertise vacancies. We only focus on non-autarchic equilibria, and, among them, on interior equilibria.

\(^6\)Details of the calculations can be found in the appendix, in step 1 of the proof of proposition 1.
Consequently, a change in \( \sigma \) has two effects on the return to schooling. First, \( \sigma \) raises the return to ability in the high-productivity sector and thus raises the return to schooling. This is a standard stabilizer effect, according to which incentives to schooling decrease as the number of individuals who get an education raises. Secondly, \( \sigma \) raises the return to ability in the low-skill sector and thus lowers the return to schooling. This is a multiplier effect according to which the incentives to schooling increase when the number of individuals who get high education raises. The sufficient condition for uniqueness (3.5) states that the latter multiplier effect should not be too strong compared to the former stabilizer effect. This is the case if the elasticity \( \varepsilon (\sigma) \) of the return to schooling \( \psi_h (\sigma) - \psi_l (\sigma) \) with respect to \( \sigma \) is larger than -1.

3.3. Equilibrium consequences of the composition effect

We close this section by studying the equilibrium implications of the composition effect. We emphasize three points. The first two points focus on the long-run consequences of a change in the subsidy \( S \), and of changes in sector-specific productivity \( A_h \) and \( A_l \), when the equilibrium is unique. The third point focuses on implications when there are multiple equilibria.

*Education policy and unemployment.* Our model displays a negative structural relationship between tightness in each sector and the share of workers with high education. Consider a rise in the subsidy \( S \). The threshold ability \( \sigma^* \) falls, that is \( \frac{d\sigma^*}{dS} < 0 \), which implies \( \frac{d\theta^*_i}{dS} < 0 \). In each sector, firms expect a lower average profitability of a match, which causes a drop in tightness. The unemployment rate \( u^*_i \) rises in both education groups\(^7\).

*Changes in sector-specific productivity.* Consider changes in the productivity of each sector \( A_h \) and \( A_l \). Let \( A_i \) denote the technological parameter of sector \( i \), and \( A_{-i} \) that of the other sector. Let also \( \Theta_i (\sigma) \equiv \Theta_i (A_i, \sigma) \) to make the dependence of function \( \Theta_i \) vis-à-vis \( A_i \) explicit. From equation (3.2), we get

\[
\frac{d\theta^*_i}{dA_i} = \frac{\partial \Theta_i (A_i, \sigma^*)}{\partial A_i} + \frac{\partial \Theta_i (A_i, \sigma^*)}{\partial \sigma} \frac{d\sigma^*}{dA_i}
\]

(3.8)

\[
\frac{d\theta^*_i}{dA_{-i}} = \frac{\partial \Theta_i (A_i, \sigma^*)}{\partial \sigma} \frac{d\sigma^*}{dA_{-i}}
\]

(3.9)

Sector-specific productivity exerts a direct positive effect on sector-specific tightness, and an indirect effect via changes in the composition of education groups. A rise in \( A_h \) (e.g. a

\(^7\)This result complements those of Acemoglu (1999) and Albrecht and Vroman (2002) where an increase in education-specific unemployment rates can be accounted for by an *exogenous* rise in the supply of skills.
skill-biased technological change) improves the return to schooling, and, therefore, raises the number of workers with higher education (\(i.e. \frac{d\sigma^*/dA_h}{dA_h} < 0\)). This effect leads to a fall in the mean ability of the low education group. Unemployment goes up for the less educated, and goes down for the others. A fall in \(A_l\) (e.g. increased competition from low-wage economies) reduces the cut-off \(\sigma\) (\(i.e. -\frac{d\sigma^*/dA_l}{dA_l} < 0\)), and raises unemployment at all education levels. Therefore, a rise in \(A_h\) cannot account for a rise in unemployment for both education groups\(^8\), contrary to a fall in \(A_l\) or a rise in the level of the subsidy \(S\).

*Multiple equilibria.* As set above, the schooling decision originates two conflicting effects on the return to schooling: a multiplier effect through the impact of \(\sigma\) on the return to ability in the low-skill sector, and a stabilizer effect through its effect on the return to ability in the high-skill sector. When the condition (3.5) does not hold, the multiplier effect may dominate the stabilizer effect, originating multiple equilibria\(^9\). The equilibria with the largest share of educated have higher unemployment rates for all education groups. When multiple equilibria exist, small changes in the economic environment can have important qualitative effects. For instance, a less stringent education policy can make the economy topple from the “low skill, low education-specific unemployment rates” equilibrium, to the “high skill, high education-specific unemployment rates” equilibrium. However, this move is not socially desirable as Section 4 makes it clear.

4. Education policy

In this section, we compare private and social returns to schooling. We show that private returns are always larger than social returns, owing to the fact that individuals do not internalize the effect of their schooling choice on firms’ incentives to enter each sector. This gives rise to over-education.

We now show this point formally. For this purpose, we consider the problem of a benevolent social planner. When the rate of time preference \(\rho\) tends to 0, the social planner’s objective is to maximize stationary consumption (see Hosios, 1990, Pissarides, 1990), \(i.e.\) output net of search and schooling costs. The planner chooses the education

\(^8\)Empirical evidence of the rise of unemployment for high and low educated can be found in the literature, e.g. Nickell and Bell (1995), Manacorda and Petrongolo (1999). These papers also discuss the relative importance of shifts in supply and demand for skills.

\(^9\)Other cases of increasing returns to schooling due to coordination failures can be found in the matching literature. See e.g. Laing et al (1995), McKenna (1996), Saint-Paul (1996), Burdett and Smith (2002).
subsidy $S$. to maximize aggregate welfare:

$$P = \sum_{i=h,l} (Y_i - \gamma v_i) - \delta (1 - \Phi (\sigma)) C \quad (4.1)$$

where $Y_i$ is flow output in sector $i$. Welfare is the sum of output net of search costs, in each sector, minus schooling costs times the stationary number of students of a given cohort. Given $v_i = \theta_i u_i$, $u_i = \frac{\delta}{m(\theta_i) + \delta} \sigma_i$, $Y_i = (\sigma_i - u_i) A_i E_i (\sigma)$, (4.1) can be re-written

$$P = \sum_{i=h,l} \left( \frac{m(\theta_i)}{m(\theta_i) + \delta} \sigma_i A_i E_i (\sigma) - \delta \gamma \frac{\theta_i}{m(\theta_i) + \delta} \sigma_i \right) - \delta (1 - \Phi (\sigma)) C \quad (4.2)$$

In the standard matching model without educational choices, it is well-known that the decentralized allocation is generally inefficient unless the so-called Hosios condition holds. Therefore, education decisions taken in an otherwise inefficient environment are likely to be inefficient. To highlight the novel inefficiency due to self-selection in education, we assume the planner has only one tool to alter the market outcome: the schooling subsidy $S$.

The social planner maximizes $P$ subject to the free-entry condition (3.2), the self-selection condition (3.3). Thus,

$$\hat{S} \in \arg \max \left\{ \sum_{i=h,l} \sigma_i \mathbb{E} U_i (a) - (1 - \Phi (\sigma)) C \right\} \quad (4.3)$$

The objective is the sum of the utilities expected by a given cohort of entrants, that is the number of individuals of each type times their average utility, minus schooling costs.

**Proposition 2** Let $S_{\min} \equiv C - \lim_{\sigma \to 1} \{ \psi_h (\sigma) - \psi_l (\sigma) \}$. Suppose a unique interior equilibrium exists for all $S \in (S_{\min}, C)$. The planner chooses $\hat{S} < 0$, i.e. overeducation takes place.

The optimal policy is to set a tax on education to deter low-ability individuals from entering the high-productivity sector. This is true irrespective of bargaining power, schooling cost and technology. The self-selection process is therefore inefficient. Why does such inefficiency occur? A fall in $\sigma$ deteriorates job seekers’ search prospects in each submarket,

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10This condition states the workers’ bargaining power should be equal to the elasticity of the matching technology with respect to the number of vacancies.

11This departs from the “usual” problem where the planner chooses an allocation of the workforce subject to matching frictions and feasibility constraints. This maximisation problem is therefore more constrained than the usual problem, and the optimal subsidy is likely to be more constrained efficient.
and leads to a welfare loss. The welfare loss cannot be compensated by the gains accruing to the threshold individual, because they are exactly offset by the schooling costs.

Here is another way of looking at the same thing. Overeducation means that society would be better-off if the least able among the educated had not educated themselves in the first place. The social marginal productivity of those individuals is negative. From the social point of view, an individual of ability \( a \) should acquire education only if

\[
\frac{m_h}{m_h + \delta} \left( \frac{A_h a}{\delta} - \frac{\gamma \theta_h}{m_h} \right) \geq C + \frac{m_l}{m_l + \delta} \left[ \frac{A_l a}{\delta} - \frac{\gamma \theta_l}{m_l} \right]
\]

(4.4)

The LHS is the discounted value of the output flow net of average search costs multiplied by the employment rate in the high-skill sector. The RHS is the sum of schooling costs plus the opportunity cost. Opportunity costs account for the discounted value of production in the low-productivity sector minus search costs multiplied by the employment rate in the low-productivity sector. With \( C = 0 \) and \( A_l = 0 \), condition (4.4) reduces to

\[
\frac{A_h a}{\delta} \geq \frac{\gamma \theta_h}{m_h} > 0
\]

(4.5)

A person should only invest in education if the discounted value of production outweighs the search cost involved to recruit such a worker. In the decentralized economy, condition (4.5) can never be satisfied because \( C = 0 \) and \( A_l = 0 \) imply \( \sigma = 0 \). From the social point of view, it may be worth deterring low ability individuals from investing in education, as their participation to the high-productivity sector generates insufficient benefits compared to the costs involved. From firms’ point of view however, it is never worth rejecting the application of these low ability workers to high-productivity jobs, though the discounted output flow generated by these workers is below the average cost they incur to recruit such a worker. This is due to free entry, which implies that the value of continuing search is nil; whatever the firm may decide, the cost of advertising a vacancy is sunk. It is therefore never optimal to reject the application of low ability candidates. As all contacts lead to matches, no decentralized mechanism is at play to deter the participation of low ability individuals to the high-productivity sector. In the absence of collective agreements to set minimum standards in the market for skills, there is too little self-selection. A possible alternative to such market failures is therefore a more stringent education policy.

Alternatively, suppose workers are equally productive in both sectors \( (A_h = A_l) \). At first glance, education is useless because it does not alter workers’ productivity. Though, while the least able remain uneducated, it is still worth investing in education for some workers, namely the ablest who would benefit from a better job-finding rate in the other
sector. When the schooling cost $C$ tends to 0, then, the whole population acquires education. According to Proposition 2 too many workers educate, though education is free: a single, high-skill market where the whole population searches for a job is inefficient because the least able workers are not worth the average amount of search and recruitment spendings incurred by firms. Segmentation of the labour market is therefore an optimal response to this market failure, and the amount of resources spent in the education system is the cost of this efficient segmentation. Hence, although education does not alter individual productivity, it is still a socially valuable activity\textsuperscript{12}.

Overeducation is related to the nature of the matching process. In a frictionless economy, the assignment is always efficient. Education does not create externalities: an increase in the share of high educated obviously alters average performances within each education group, but does not affect an individual’s wage. By contrast, when searching is costly and matching takes time, the incentive to acquire education is too high for low ability individuals, which translates into lower job and wage opportunities for both educated and uneducated workers. Yet, there is an important assumption in our paper: there are two separate matching sectors, and educated workers direct their search towards high-productivity occupations. Alternative specifications of the matching process have been considered in the literature\textsuperscript{13}: search may be undirected (see e.g. Acemoglu, 1999, and Albrecht and Vroman, 2002), or skilled workers may poach on unskilled jobs (see e.g. Gautier, 2002). Adopting those different specifications would of course alter our results, but our main mechanism would still apply: an (endogenous) increase in the share of higher educated reduces the average ability of both the low and the high educated, which tends to deteriorate their search prospects.

5. Concluding comments

This paper emphasizes a simple mechanism of overeducation in the presence of search frictions in the labour market. Our argument is related to market segmentation, workers’ heterogeneity and self-selection in education. When schooling is costly and workers are heterogenous in ability, only the ablest choose to acquire education, i.e. self-selection in education takes place. A fall in the threshold ability involves a composition effect: the

\textsuperscript{12}This makes an important difference with the signalling approach to overeducation (Arrow, 1973, and Spence, 1973), where education is a socially wasteful activity to the extent that it consumes more resources than it raises productivity.

\textsuperscript{13}However, the structure by skills of the population is exogenous in those models.
ablest among the former uneducated now become the least able among the educated. As a result, the average productivity of the economy rises, but in both education groups, the mean ability falls. This composition effect alters firms’ incentives to enter each sector. This gives rise to an important externality responsible for overeducation: workers consider their private gains from schooling, i.e. wage and employment perspectives, but not the impact of their decisions on job creation in each sector. The private return to education therefore exceeds the social one, and the optimal policy is to set a tax on education.

It goes without saying that our framework is parsimonious and leaves much of the real world out. There are, of course, a large number of reasons why there could be undereducation. Under borrowing constraints, high ability individuals from poor families may under-invest in education, and the society could find it beneficial to grant a subsidy to these students. Besides, in a high unemployment world, education improves the prospects of some economically disadvantaged groups. This can be socially valuable in various ways which have not been considered in our framework, for instance by reducing the likelihood for those groups to get involved in criminal activities. Finally, it is often emphasized that human capital externalities raise output at the aggregate level, and for this reason the social return to education may exceed the private one. Incorporating these features into our framework is on our research agenda.
APPENDIX

Proof of proposition 1 An interior equilibrium is a vector \((\theta^*_h, \theta^*_l, \sigma^*)\) which satisfies (3.2) for \(i = h, l\) and (3.3) with \(\sigma^* \in (0, 1)\). Consider the function \(\Lambda : (0, 1) \to \mathbb{R}\) such that

\[
\Lambda (\sigma) = [\psi_h (\sigma) - \psi_l (\sigma)] \sigma - (C - S) \tag{.1}
\]

\((\sigma^*, \theta^*_h, \theta^*_l)\) is an interior equilibrium iff \(\Lambda (\sigma^*) = 0\), \(\sigma^* \in (0, 1)\), \(\theta^*_h = \Theta_h (\sigma^*)\) and \(\theta^*_l = \Theta_l (\sigma^*)\).

(i) We proceed in two steps. First, we detail the properties of functions \(\Theta_l\) and \(\Theta_h\). Secondly, we show condition (3.4) implies there exists \(\sigma^* \in (0, 1)\) such that \(\Lambda (\sigma^*) = 0\) and \(\sigma^* \in (0, 1)\). Consider the pair of functions \((f_h, f_l), f_i : (0, \infty) \times (0, 1) \to \mathbb{R}, i = h, l\), such that

\[
f_i (\theta, \sigma) = (r + \beta m (\theta)) \frac{\theta}{m (\theta)} - (1 - \beta) \frac{A_i E_i (\sigma)}{\gamma} \tag{.2}
\]

where \(E_i (\sigma) \equiv \int_0^\sigma a \frac{\phi (a)}{\Phi (\sigma)} da\) and \(E_h (\sigma) \equiv \int_0^1 a \frac{\phi (a)}{1 - \Phi (\sigma)} da\).

Step 1. a) For all \(\sigma \in (0, 1)\), there exists a unique \(\theta_i \equiv \Theta_i (\sigma) > 0\) such that \(f_i (\theta_i, \sigma) = 0\). Moreover, b) \(\Theta_h (\sigma) > \Theta_l (\sigma)\) for all \(\sigma \in (0, 1)\), c) \(\Theta_i\) is strictly increasing in \(\sigma\), d) \(\lim_{\sigma \to 1} \Theta_h (\sigma) = \tilde{\theta}_h > \lim_{\sigma \to 0} \Theta_h (\sigma) = \bar{\theta}_h > \lim_{\sigma \to 1} \Theta_l (\sigma) = \tilde{\theta}_l > \lim_{\sigma \to 0} \Theta_l (\sigma) = \bar{\theta}_l = 0\).

Proof. a) For all \(\sigma \in (0, 1)\), \(0 < E_i (\sigma) < 1\). In addition, \(f_i\) is continuously differentiable. Using L’Hôpital’s rule,

\[
\lim_{\theta \to 0} \frac{\theta}{m (\theta)} = \lim_{\theta \to 0} \frac{1}{m' (\theta)} = 0 \tag{3.3}
\]

since the Inada conditions on the matching technology imply \(\lim_{\theta \to 0} m' (\theta) = \infty\). Therefore,

\[
\lim_{\theta \to 0} f_i (\theta, \sigma) = - (1 - \beta) A_i E_i (\sigma) / \gamma < 0 \tag{.4}
\]

Moreover,

\[
\lim_{\theta \to \infty} f_i (\theta, \sigma) = \infty \tag{.5}
\]

The uniqueness of the implicit function \(\Theta_i\) is implied by

\[
\frac{\partial f_i (\theta, \sigma)}{\partial \theta} = r \frac{m (\theta) - \theta m' (\theta)}{[m (\theta)]^2} + \beta \frac{(1 - \alpha (\theta)) r + \beta m (\theta)}{m (\theta)} > 0 \tag{.6}
\]

since \(\alpha (\theta) \equiv \theta m' (\theta) / m (\theta) \in (0, 1)\).
b) comes from $E_h (\sigma) > E_i (\sigma)$ for all $\sigma \in (0, 1)$, and $A_h > A_l$. c) comes from the implicit function theorem and the fact that $E_i$ is strictly increasing in $\sigma$, for $i = h, l$. Indeed,

$$\frac{d\Theta_i (\sigma)}{d\sigma} = - \frac{\partial f_i (\Theta_i (\sigma), \sigma) / \partial \sigma}{\partial f_i (\Theta_i (\sigma), \sigma) / \partial \theta} \quad (7)$$

where

$$\frac{\partial f_i (\theta, \sigma)}{\partial \sigma} = - (1 - \beta) A_i E'_i (\sigma) / \gamma \text{ for all } \theta > 0 \quad (8)$$

Making use of $f_i (\Theta_i (\sigma), \sigma) = 0$, we get

$$\frac{d\Theta_i (\sigma)}{d\sigma} = \frac{r + \beta m (\Theta_i (\sigma))}{(1 - \alpha (\Theta_i (\sigma))) \beta} \Theta_i (\sigma) \frac{dE_i (\sigma) / d\sigma}{E_i (\sigma)} > 0 \quad (9)$$

because

$$\frac{dE_h (\sigma)}{d\sigma} = \frac{\phi (\sigma)}{1 - \Phi (\sigma)} [E_h (\sigma) - \sigma] > 0 \quad (10)$$

$$\frac{dE_l (\sigma)}{d\sigma} = \frac{\phi (\sigma)}{\Phi (\sigma)} [\sigma - E_l (\sigma)] > 0 \quad (11)$$

d) is implied by the properties of the matching technology (the same properties we used to establish the existence and uniqueness of functions $\Theta_h$ and $\Theta_l$). The limit values $\bar{\theta}_h$ and $\bar{\theta}_l$ are derived from

$$\lim_{\sigma \to 1} f (\bar{\theta}_i, \sigma) = 0 \quad (12)$$

where $\lim_{\sigma \to 1} E_h (\sigma) = 1$ and $\lim_{\sigma \to 1} E_l (\sigma) = \mathbb{E} (a)$, the unconditional mean ability in the whole population. The limit $\bar{\theta}_h$ and $\bar{\theta}_l \equiv 0$ are derived from

$$\lim_{\sigma \to 0} f (\bar{\theta}_i, \sigma) = 0 \quad (13)$$

where $\lim_{\sigma \to 0} E_h (\sigma) = \mathbb{E} (a)$ and $\lim_{\sigma \to 0} E_l (\sigma) = 0$. The fact that $\bar{\theta}_h > \bar{\theta}_l$ results from $A_h > A_l$.

Step 2. Conclusion.

>From step 1, the function $\psi_i (\sigma) \sigma$ is strictly increasing in $\sigma$, from $\lim_{\sigma \to 0} \psi_i (\sigma) \sigma = 0$ to $\lim_{\sigma \to 0} \psi_i (\sigma) \sigma = \frac{\beta m (\bar{\theta}_i)}{r + \beta m (\bar{\theta}_i)} A_i > 0$, and $\psi_h (\sigma) \sigma > \psi_i (\sigma) \sigma$ for all $\sigma > 0$. Then,

$$\lim_{\sigma \to 1} \Lambda (\sigma) = -(C - S) \text{ and } \lim_{\sigma \to 1} \Lambda (\sigma) = \lim_{\sigma \to 1} \{\psi_h (\sigma) - \psi_i (\sigma)\} - (C - S).$$

Claim (i) follows from the continuity of $\Lambda$.

(ii) As functions $\Theta_h$ and $\Theta_l$ are differentiable, it is also the case that function $\Lambda$ is differentiable. It comes

$$\Lambda' (\sigma) = (1 + \varepsilon (\sigma)) [\psi_h (\sigma) - \psi_l (\sigma)] \quad (14)$$
Condition (3.4) implies existence. Since $\psi_h(\sigma) > \psi_l(\sigma)$, condition (3.5) implies uniqueness. ■

**Proof of proposition 2** Recall the free-entry conditions (3.2) define two implicit functions $\Theta_h$ and $\Theta_l$ of $\sigma$, which properties are provided in step 1 of the proof of proposition 1. Let $\sigma^* \equiv \sigma^*(S,.)$ be denoted by $\sigma$. The objective is continuously differentiable in $C$ and $S \in (S_{\text{min}}, C)$. Taking the derivative of $P$ with respect to $S$ yields:

\[
\frac{dP}{dS} = \sum_i A_i \frac{\beta m_i}{\delta + \beta m_i} \left[ \frac{\delta}{\delta + \beta m_i} \frac{m_i'}{m_i} \frac{d\sigma_i}{d\sigma} \sigma_i E_i(\sigma) + \frac{d(\sigma_i E_i(\sigma))}{d\sigma} \right] \frac{d\sigma}{dS}
\]

(15)

where $m_i \equiv m(\Theta_i(\sigma))$ and $m_i' \equiv m'(\Theta_i(\sigma))$. But $\sigma_i E_i(\sigma) = \int_0^\sigma a \phi(a) \, da$ and $\sigma_h E_h(\sigma) = \int_\sigma^1 a \phi(a) \, da$. Consequently,

\[
\frac{d(\sigma_i E_i(\sigma))}{d\sigma} = \sigma \phi(\sigma)
\]

(16)

\[
\frac{d(\sigma_h E_h(\sigma))}{d\sigma} = -\sigma \phi(\sigma)
\]

(17)

Moreover, we know from (3.3) that

\[
C = \frac{\beta m_h}{\delta + \beta m_h} \frac{A_h}{\delta} \sigma - \frac{\beta m_l}{\delta + \beta m_l} \frac{A_l}{\delta} \sigma + S
\]

(18)

Therefore,

\[
\frac{dP}{dS} = \left\{ \sum_i A_i \frac{\beta m_i}{\delta + \beta m_i} \frac{\delta}{\delta + \beta m_i} \frac{m_i'}{m_i} \frac{d\Theta_i}{d\sigma} \sigma_i E_i(\sigma) + \delta \phi(\sigma) S \right\} \frac{d\sigma}{dS}
\]

(19)

which has the opposite sign of the term between brackets since the uniqueness of equilibrium implies $d\sigma/dS < 0$. However, the properties of the matching technology imply $m_i' > 0$, while we established previously that $d\Theta_i/d\sigma > 0$. Therefore, $dP/dS < 0$ whenever $S \geq 0$. Proposition 2 follows. ■
References


